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Dynamic models of wind farms with fixed speed wind turbines

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Abstract

The increasing wind power penetration on power systems requires the development of adequate wind farms models for representing the dynamic behaviour of wind farms on power systems. The behaviour of a wind farm can be represented by a detailed model including the modelling of all wind turbines and the wind farm electrical network. But this detailed model presents a high order model if a wind farm with high number of wind turbines is modelled and therefore the simulation time is long. The development of equivalent wind farm models enables the model order and the computation time to be reduced when the impact of wind farms on power systems is studied. In this paper, equivalent models of wind farms with fixed speed wind turbines are proposed by aggregating wind turbines into an equivalent wind turbine that operates on an equivalent wind farm electrical network. Two equivalent wind turbines have been developed: one for aggregated wind turbines with similar winds, and another for aggregated wind turbines under any incoming wind, even with different incoming winds.

The proposed equivalent models provide high accuracy for representing the dynamic response of wind farm on power system simulations with an important reduction of model order and simulation time compare to that of the complete wind farm modelled by the detailed model.

Keywords: Aggregated wind turbines; Equivalent model; Fixed speed wind turbines; Wind farms

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1. Introduction

As result of environmental concern, the penetration of renewable power on power systems is increasing and thus the impact of conventional electrical generation on environment is being minimized. This development of electrical generation from renewable sources is due to the absence of harmful emissions on the environment and the infinite availability of the energy used to be converted into electrical energy.

Wind energy is one way of electrical generation from renewable sources that uses wind turbines to convert the energy contained in flowing air into electrical energy. Wind power is the world's fastest growing energy source with a average growth over the past 5 years of 26% and a foreseeable penetration 12% of global electricity demand by 2020 [1]. This important growth has been achieved by concentrating a large number of wind turbines in wind farms for a better exploitation of regions with good wind resources. As result of increasing wind farms penetration on power systems, the wind farms begin to influence power system and the power system operators are concerned about the behaviour of power systems with wind farms. This justifies the need for development of adequate wind farms models for representing the response of wind farm, evaluating their influence and thus improving the planning and exploitation of electrical network.

Actually, one of the used wind farm concepts in power systems is based on fixed speed wind turbines with directly grid coupled squirrel cage induction generator connected to the wind turbine rotor through gearbox. This generator presents very small rotational speed variations because of the only speed variations that can occur are changes in the rotor slip, and therefore these wind turbines are considered to operate at fixed speed. A squirrel cage induction generator consumes reactive power, and therefore compensating capacitors are added to generate the induction generator magnetizing current, thus improving the power factor.

The wind farms with fixed speed wind turbines are composed of a large number of wind turbines with directly grid coupled squirrel cage induction generator and compensating capacitors, operating on an internal electrical network (lines and transformers) to connect the wind farm to the grid.

The dynamic behaviour of wind farms has been usually represented by a detailed model, including the modelling of all wind turbines and the internal electrical network [2–4]. But this detailed model presents a high order model if a wind farm with high number of wind turbines is modelled and therefore the simulation time is long, because of the excessive number of equations to be computed. When simulating wind farms with fixed speed wind turbines on power systems, the complexity of the wind farm model and the simulation time can be reduced by equivalent models.

Equivalent wind farm models have been developed by aggregating wind turbines with identical incoming wind speed into an equivalent wind turbine, experiencing that wind as incoming wind, and operating on an equivalent wind farm electrical network [5–8]. These equivalent wind farms present so many equivalent wind turbines as wind turbines groups with identical winds. In large scale power systems, it is usual to consider identical winds incident on all the wind turbines of wind farm and therefore the wind farm can be represented by a single equivalent wind turbine [9–11].

For wind turbines under different incoming wind speeds, Slootweg and Kling [12,13] propose the aggregation of the mechanical powers of individual wind turbines, calculated by a simplified model of each wind turbine and the incoming wind speed. This aggregated mechanical power is applied to an equivalent generation system obtained from aggregating the generation systems of wind turbines that presents an equivalent induction generator and constant equivalent compensating capacitors.

Other equivalents wind farm models have been proposed by Castro and Ferreria [14] by applying singular perturbations theory, and Kanellos and Hatziargyriou [15] by system identification theory.

The main objective of this work is to contribute to the topic of fixed speed wind farm modelling by the development of reduced models for representing the dynamic behaviour on power systems. This paper presents equivalent models of wind farms with fixed speed wind turbines that are based on aggregating wind turbines into an equivalent wind turbine operating on an equivalent wind farm electrical network. Two equivalent wind turbines have been developed, both including an equivalent generation system but: (1) one by using an equivalent wind incident on an equivalent mechanical system for aggregated wind turbines with similar winds, (2) another by using each wind incident on a simplified model of each individual wind turbine for aggregated wind turbines under any wind. These equivalent models have been validated by simulations in which the responses of equivalent and detailed wind farms have been compared.

The paper is structured as follows. Section 2 describes the detailed wind farm model, and Section 3 presents the equivalent models of aggregated wind turbines and wind farm electrical network. Section 4 depicts the simulation results obtained by comparing the equivalent and detailed wind farm models both for a normal operation and a grid disturbance. Finally, conclusions are presented in Section 5.

2. Detailed wind farm model

The dynamic response of a wind farm, as the configuration shown in Fig. 1, can be described by a detailed model, including the modelling of all the wind turbines and the wind farm electrical network.

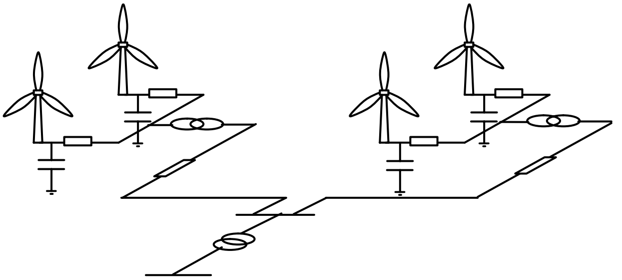


Fig. 1. Wind farm with fixed speed wind turbines.

2.1. Wind turbine model

The fixed speed wind turbine is composed of a directly grid coupled squirrel cage induction generator connected to the wind turbine rotor through a gearbox. The rotor limits the power extracted from the wind by using the stall effect (stall regulated wind turbine) or by controlling the blade pitch angle (pitch regulated wind turbine) to decrease the rotor aerodynamic efficiency for high wind speeds and thus limiting the mechanical power extracted from the wind.

For power systems simulations, the behaviour of wind turbine can be represented by modelling the rotor, drive train and generation system model with squirrel cage induction generator and compensating capacitors.

Applying the actuator disk theory [16,17], the rotor model provides the aerodynamic torque extracted from the wind by the following equation

$$T_{\mathrm{w}} = \frac{1}{2}\rho A u^2 \frac{C_{\mathrm{p}}}{\lambda} \tag{1}$$

where ρ (kg/m^3) is the air density, A (m^2) is the rotor disk area, u (m/s) is the wind speed and C_{p} is the power coefficient which is a function of the tip speed ratio λ and the blade pitch angle θ for pitch regulated wind turbines, $C_{\mathrm{p}}(\lambda,\theta)$, and only function of the tip speed ratio for stall regulated wind turbines, $C_{\mathrm{p}}(\lambda)$. The tip speed ratio is defined as

$$\lambda = \frac{\omega_{\mathrm{b}} R}{u} \tag{2}$$

where ω_{b} (rad/s) is the blade angular speed and R (m) is the blade length.

In power systems simulations, the drive train model is usually represented by two masses [6,18] (as shown in Fig. 2), the first mass stands for the wind turbine rotor (blades, hub and low-speed shaft), while the second mass stands for generator rotor (high-speed shaft). The equations of the model, expressed in per unit, are

$$T_{\mathrm{w}} - T_{\mathrm{m}} = J_{\mathrm{r}} \frac{\mathrm{d}\omega_{\mathrm{r}}}{\mathrm{d}t} \tag{3}$$

$$T_{\mathrm{m}} = D_{\mathrm{mc}}(\omega_{\mathrm{r}} - \omega_{\mathrm{g}}) + K_{\mathrm{mc}} \int (\omega_{\mathrm{r}} - \omega_{\mathrm{g}})\mathrm{d}t \tag{4}$$

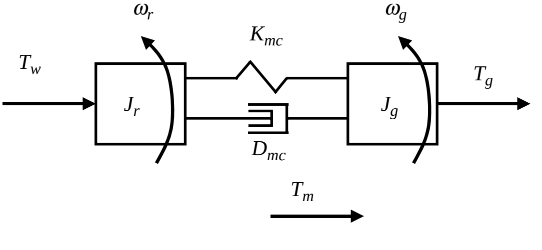


Fig. 2. Drive train model.

$$T_m - T_g = J_g \frac{d\omega_g}{dt} \tag{5}$$

where ω_r, J_r, ω_g, J_g are the speed and inertia of the wind turbine rotor and generator, T_m is the generator mechanical torque, T_g is the generator electrical torque and K_{mc} and D_{mc} are the stiffness and damping of the mechanical coupling.

The squirrel cage induction generator is represented by means of the well-known third-order transient state induction machine model with stator transients neglected, as usual for representing the induction generator on power systems transient stability studies [18,19]. The differential equations of per unit generator model are expressed as

$$\frac{de'_d}{dt} = -\frac{1}{T'_0}[e'_d - (X_s - X'_s)i_{qs}] + s\omega_s e'_q \tag{6}$$

$$\frac{de'_q}{dt} = -\frac{1}{T'_o}[e'_q + (X_s - X'_s)i_{ds}] - s\omega_s e'_d \tag{7}$$

where e'_d and e'_q are the internal voltage components of the equivalent Thèvenin, and i_{ds} and i_{qs} the stator current components.

The third differential equation is the generator mechanical Eq. (5) included in the drive train model. The generator electrical torque is calculated as

$$T_g = e'_d i_{ds} + e'_q i_{qs} \tag{8}$$

And the active and reactive powers of induction generator can be expressed by

$$P_e = e'_d i_{ds} + e'_q i_{qs} \tag{9}$$

$$Q_e = e'_d i_{qs} - e'_q i_{ds} \tag{10}$$

where T'_o is the transient open circuit time constant and X'_s is the transient reactance, expressed as

$$T'_o = \frac{X_{\sigma r} + X_m}{\omega_s R_r} \qquad X'_s = (X_{\sigma s} + X_m) - \frac{X_m^2}{X_{\sigma r} + X_m} \tag{11}$$

where R_s and R_r are the stator and rotor resistances, $X_{\sigma s}$ and $X_{\sigma r}$ are the stator and rotor leakage reactances, and X_m is the magnetizing reactance.

Considering that the fixed speed wind turbine uses local compensating capacitors, the total current injected by the wind turbine at the generation node is given by

$$i_{dg} = i_{ds} + i_{dc} = i_{ds} + \frac{1}{X_c} u_{qg} \tag{12}$$

$$i_{qg} = i_{qs} + i_{qc} = i_{qs} - \frac{1}{X_c} u_{dg} \tag{13}$$

where X_c is the reactance of the compensating capacitors, i_{dc} and i_{qc} are the capacitors current components, u_{dg} and u_{qg} are the wind turbine generation voltage components.

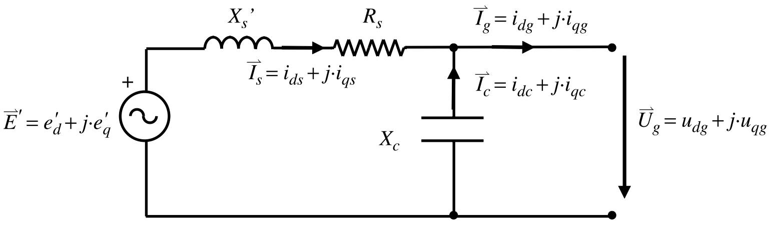


Fig. 3. Equivalent circuit of fixed speed wind turbines.

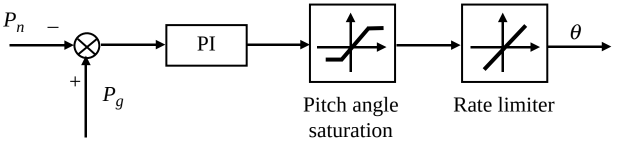


Fig. 4. Control scheme of pitch regulated wind turbines.

The active and reactive powers delivered by the wind turbine at the coupling point to wind farm can be expressed as

$$P_g = u_{dg} i_{dg} + u_{qg} i_{qg} \tag{14}$$

$$Q_g = u_{qg} i_{dg} - u_{dg} i_{qg} \tag{15}$$

The generation system, including the induction generator and compensating capacitors, can be represented by the equivalent circuit presented in Fig. 3.

If the wind turbines are pitch regulated, the model includes the pitch angle controller for limiting the generated power to rated power in above rated wind speeds. The control scheme implemented in this paper uses a PI controller that determinates the pitch angle reference to the pitch angle actuator by the power error. The pitch angle actuator has been modelled including saturation and rate limiter, as depicted in Fig. 4.

2.2. Wind farm electrical network model

The wind farm electrical network has been modelled by the static model of electric lines and transformers, represented by constant impedance, as usual for power systems simulations [19]. This implies that only fundamental frequency components are taken into account and subtransient phenomena are neglected.

3. Equivalent wind farm models

When simulating wind farms with large number of wind turbines on power system, the usual way of reducing the wind farm model is based on aggregating identical wind turbines into an equivalent wind turbine that operates on an equivalent network of aggregated wind turbines, and thus an equivalent wind farm model is derived.

The aggregation of wind turbines is mathematically exact when receiving the same wind and therefore generating the same output power. But if the aggregated wind turbines present different incoming winds, theirs output powers differ and therefore the wind incident on each wind turbine must be taken account when aggregating method is applied.

Considering that wind farms can present wind turbines with similar and different incoming winds depending on the wind farm location, two equivalent wind turbines have been developed in this work: (1) equivalent wind turbine with equivalent incoming wind for aggregated wind turbines with similar winds; (2) equivalent wind turbine with a simplified wind turbine model for aggregated wind turbines under any wind, even under different incoming winds.

3.1. Equivalent wind turbine with equivalent incoming wind

Wind farms located on topographically simple terrains (e.g. smooth land or off-shore) present wind turbines organized in rows at right angles to the prevailing wind direction, with the wind turbines separated a minimum distance between three or five times rotor diameter within the same row and five or nine times between rows [20]. In these wind farms, the wind turbines belonging to the same rows usually present similar winds, whereas the winds incident on each row are different because of shadowing between wind turbines (park effect) [20]. Therefore, these wind farms present groups of wind turbines with similar winds, the wind turbines within the same row.

In this paper, an equivalent model for these wind farms is proposed by aggregating wind turbines with similar winds into an equivalent wind turbine. Thus, the equivalent wind farm present so many equivalent wind turbines as wind turbines groups with similar winds, as shown in Fig. 5.

The equivalent wind turbine presents n-times the size of individual wind turbines, and therefore a rated power equal to n-times the rated power of individual wind turbines, where n is the number of aggregated wind turbines.

The wind speeds of aggregated wind turbines can not be added since the non-linear relationship between wind speed and mechanical torque or power would introduce an error. Because of the differences between the output powers of wind turbines with similar winds are minor, an equivalent wind of the winds incident on the aggregated wind turbines can be used in the equivalent wind turbine.

If the aggregated wind turbines receive the same wind, this is the equivalent wind, but if the winds are different, the average wind of individual winds has been used as equivalent wind in this paper.

This equivalent wind is applied to the same rotor model than that of individual wind turbine, with identical rotor diameter and power coefficient, and thus obtaining the aerodynamic torque of an individual wind turbine. Supposing that all the aggregated wind turbines are operating with the equivalent wind, the aerodynamic torque of equivalent wind turbine can be approximated as

$$T_{w,e} = nT_{w,i} \tag{16}$$

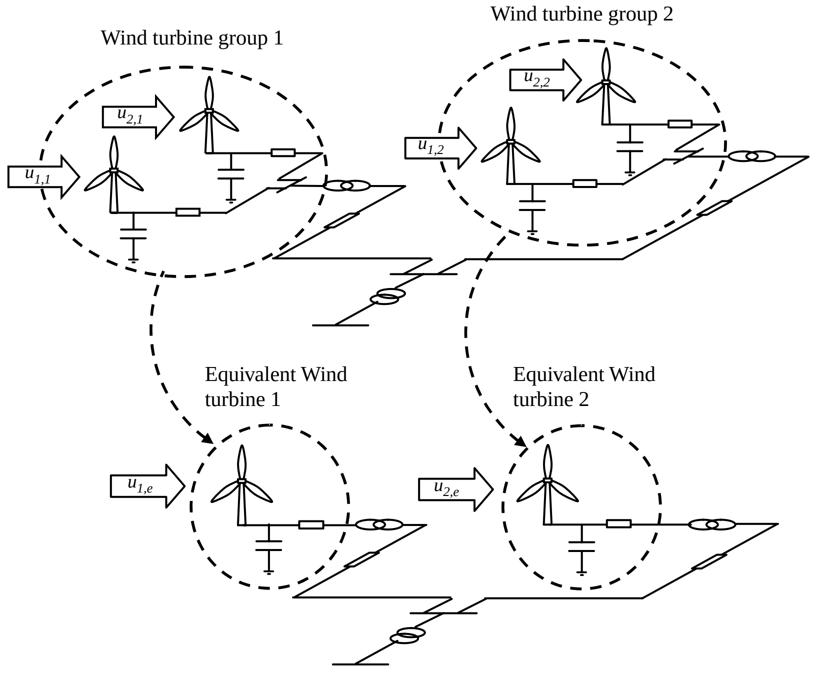


Fig. 5. Equivalent wind farm by using equivalent wind turbines for aggregated wind turbines with similar winds.

where i subindex corresponds to individual wind turbine and e subindex to equivalent wind turbine, respectively.

This equivalent aerodynamic torque is applied to an equivalent wind turbine that presents the same drive train and generation system models than those of individual wind turbines, given by Eqs. (3)–(15) with the same mechanical and electrical parameters per unit. A scheme of the equivalent wind turbine model is shown in Fig. 6.

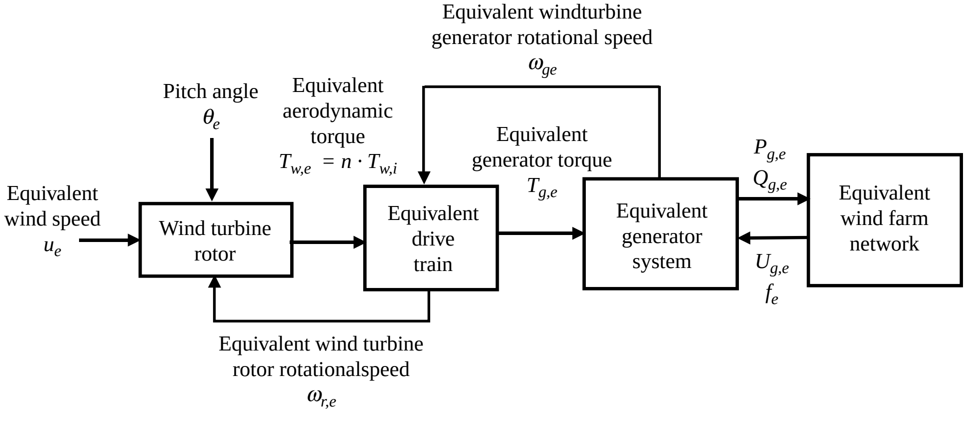


Fig. 6. Equivalent wind turbine model with equivalent incoming wind.

3.2. Equivalent wind turbine with simplified wind turbine model

Wind farms located on topographically complex terrains (e.g. mountain ladder) or with widely separated wind turbines present wind turbines with different incoming winds and therefore the output powers of individual wind turbines are different. The equivalent wind turbine described before is not valid for representing the response of aggregated wind turbines, since the equivalent wind is unable to represent the differences over generation between the aggregated wind turbines. And therefore the turbines of wind turbines with different incoming winds cannot be aggregated.

In this paper, it has been developed a new equivalent wind turbine for aggregating wind turbine under different incoming winds. This equivalent wind turbine presents an aggregated model of the generation systems and a simplified model of each individual wind turbine for approximating the operational points of each wind turbine according to the incoming wind speed. This way to aggregate wind turbines enables to develop equivalent wind farms by using equivalent wind turbines of wind turbines experiencing different incoming winds, as shown in Fig. 7.

The simplified model of stall regulated wind turbines is composed of the rotor and drive train models described by mechanical Eqs. (1)–(5), and the steady-state generator electrical torque calculated by

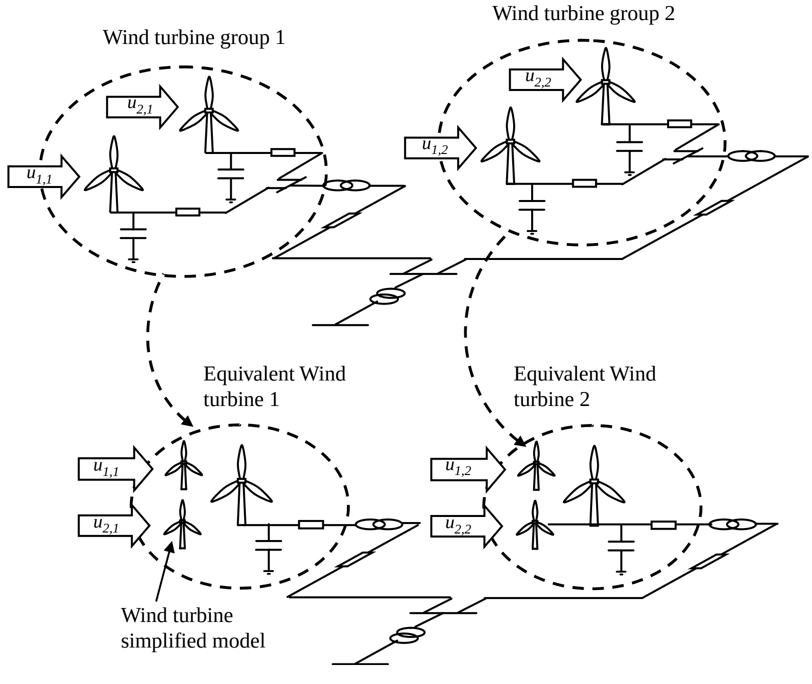


Fig. 7. Equivalent wind farm by using equivalent wind turbines for aggregated wind turbines under different incoming winds.

$$T_{\mathrm{g,i}} = \frac{3R_{\mathrm{r}}(U_{\mathrm{g,i}}/\sqrt{3})^2 s_{\mathrm{i}}}{\omega_{\mathrm{s}}((R_{\mathrm{s}}s_{\mathrm{i}} + R_{\mathrm{r}})^2 + ((X_{\sigma\mathrm{s}} + X_{\sigma\mathrm{r}})s_{\mathrm{i}})^2)} \tag{17}$$

where $U_{\mathrm{g,i}}$ is the generation voltage of the wind turbine.

If the aggregated wind turbines are pitch regulated, the simplified wind turbine model also includes the blade pitch angle controller, shown in Fig. 4.

The active power generated by each wind turbine is calculated as

$$P_{\mathrm{g,i}} = T_{\mathrm{g,i}}\omega_{\mathrm{g,i}} \tag{18}$$

The generator mechanical torques of individual wind turbines, calculated by wind turbine simplified model, are aggregated for calculating the generator mechanical torque of the equivalent wind turbine

$$T_{\mathrm{m,e}} = \sum_{i=1}^{n} T_{\mathrm{m,i}} \tag{19}$$

This generator mechanical torque is applied to an equivalent generation system that presents identical model to those of individual wind turbines, represented by Eqs. (5)–(15) with the same mechanical and electrical parameters per unit.

If the winds incident on the wind turbines are different, it appears differences over the active power and the generation voltage between the wind turbines. Considering that the reactive power of fixed speed wind turbines depends on the active power and generation voltage, the reactive power of equivalent wind turbine would differ from that of aggregated wind turbines. In this paper, it has been used compensating capacitors with variable reactance for a better approximation of reactive power of aggregated wind turbines under different incoming winds. The procedure to calculate the capacitor reactance of the equivalent wind turbine is described next.

The simplified wind turbine model includes an approximation for calculating the reactive power of each individual wind turbine, derived from the steady-state generator model (shown in Fig. 8)

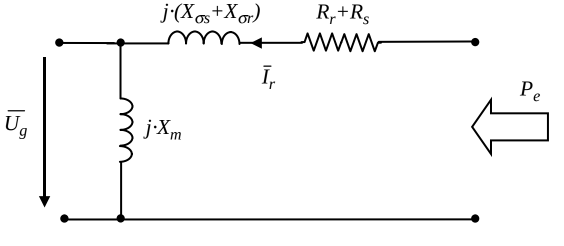


Fig. 8. Steady state induction generator model.

$$Q_{\mathrm{g,i}} = U_{\mathrm{g,i}}^2 \frac{X_{\mathrm{c}} - X_{\mathrm{m}}}{X_{\mathrm{c}} X_{\mathrm{m}}} + (X_{\sigma \mathrm{s}} + X_{\sigma \mathrm{r}}) \frac{U_{\mathrm{g,i}}^2 + 2(R_{\mathrm{r}} + R_{\mathrm{s}}) P_{\mathrm{g,i}}}{2((R_{\mathrm{r}} + R_{\mathrm{s}})^2 + (X_{\sigma \mathrm{s}} + X_{\sigma \mathrm{r}})^2)} - (X_{\sigma \mathrm{s}} + X_{\sigma \mathrm{r}}) \frac{\sqrt{(U_{\mathrm{g,i}}^2 + 2(R_{\mathrm{r}} + R_{\mathrm{s}}) P_{\mathrm{g,i}})^2 - 4 P_{\mathrm{g,i}}^2 ((R_{\mathrm{r}} + R_{\mathrm{s}})^2 + (X_{\sigma \mathrm{s}} + X_{\sigma \mathrm{r}})^2)}}{2((R_{\mathrm{r}} + R_{\mathrm{s}})^2 + (X_{\sigma \mathrm{s}} + X_{\sigma \mathrm{r}})^2)} \tag{20}$$

where the active power is calculated by the simplified wind turbine model.

The generation voltage of each individual wind turbine is obtained from the equivalent wind farm network considering the following approximation

$$U_{\mathrm{g,i}} = U_{\mathrm{c,g}} - \Delta U_{\mathrm{i}} \tag{21}$$

where $U_{\mathrm{c,g}}$ is the common node voltage of aggregated wind turbines in the equivalent wind farm and ΔU_{i} is the voltage drop in the electric line of each wind turbine ($R_{\mathrm{l,i}}$ and $X_{\mathrm{l,i}}$ are the resistance and reactance of the electric line), calculated as

$$\Delta U_{\mathrm{i}} = R_{\mathrm{l,i}} P_{\mathrm{g,i}} - X_{\mathrm{l,i}} Q_{\mathrm{g,i}} \tag{22}$$

The reactive power of aggregated wind turbines is

$$Q_{\mathrm{g,g}} = \sum_{i=1}^{n} Q_{\mathrm{g,i}} \tag{23}$$

The reactive power of equivalent wind turbine without compensating capacitors is obtained as

$$Q_{\mathrm{g,e}} = \frac{U_{\mathrm{g,e}}^2}{X_{\mathrm{m,e}}} + (X_{\sigma \mathrm{s,e}} + X_{\sigma \mathrm{r,e}}) \frac{U_{\mathrm{g,e}}^2 + 2(R_{\mathrm{r,e}} + R_{\mathrm{s,e}}) P_{\mathrm{g,e}}}{2((R_{\mathrm{r,e}} + R_{\mathrm{s,e}})^2 + (X_{\sigma \mathrm{s,e}} + X_{\sigma \mathrm{r,e}})^2)} - (X_{\sigma \mathrm{s,e}} + X_{\sigma \mathrm{r,e}}) \times \frac{\sqrt{(U_{\mathrm{g,e}}^2 + 2(R_{\mathrm{r,e}} + R_{\mathrm{s,e}}) P_{\mathrm{g,e}})^2 - 4 P_{\mathrm{g,e}}^2 ((R_{\mathrm{r,e}} + R_{\mathrm{s,e}})^2 + (X_{\sigma \mathrm{s,e}} + X_{\sigma \mathrm{r,e}})^2)}}{2((R_{\mathrm{r,e}} + R_{\mathrm{s,e}})^2 + (X_{\sigma \mathrm{s,e}} + X_{\sigma \mathrm{r,e}})^2)} \tag{24}$$

Because of the capacitors of equivalent wind turbine must generate the difference between the total reactive power of the aggregated wind turbines and the reactive power of the equivalent wind turbine without capacitors, the reactance of equivalent capacitor is given by

$$X_{\mathrm{c,e}} = \frac{U_{\mathrm{g,e}}^2}{Q_{\mathrm{g,g}} - Q_{\mathrm{g,e}}} \tag{25}$$

The equivalent wind turbine with simplified model of each individual wind turbine is presented in Fig. 9, and the simplified wind turbine model in Fig. 10.

3.3. Equivalent wind farm network

The described equivalent wind turbines operate on an equivalent wind farm network, obtained from the equivalent of lines and/or transformers of the common network of aggregated wind turbines, as shown in Figs. 5 and 7.

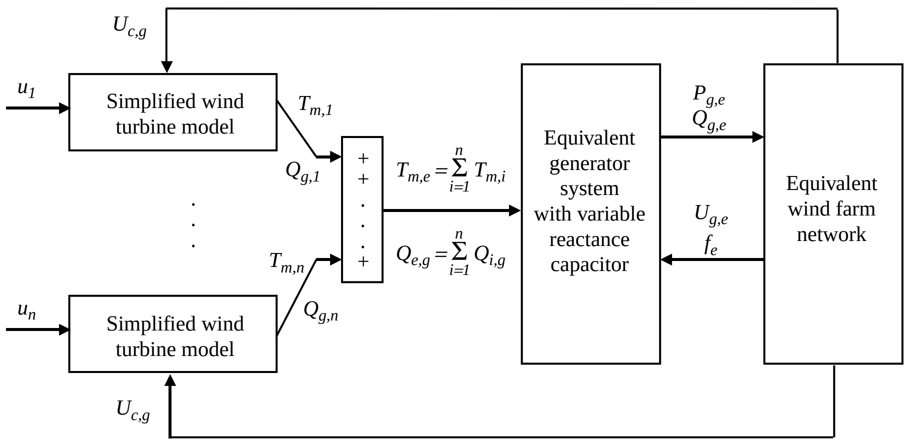


Fig. 9. Equivalent wind turbine with simplified model of each individual wind turbine.

4. Results and discussion

The equivalent models of fixed speed wind farms proposed in this paper have been verified by comparing the steady state and dynamic responses with that of the complete wind farm simulated by the detailed model. These wind farm models have been implemented and simulated by using MATLAB/Simulink [21].

4.1. Steady-state simulations

Steady-state simulations by using constant winds incident on the aggregated wind turbines have been run to obtain the application limits of the equivalent models. These winds, although representing fictitious operation conditions in the wind turbines, enable the application limits of the equivalent models to be establish according to the differences between the incoming winds.

The wind farm under consideration has two stall regulated fixed speed wind turbines of 350 kW, 660 V each, connected to the wind farm electrical network through a low voltage electric line at 660 V (LV). A transformer 20/0.66 kV boosts up the voltage and a medium

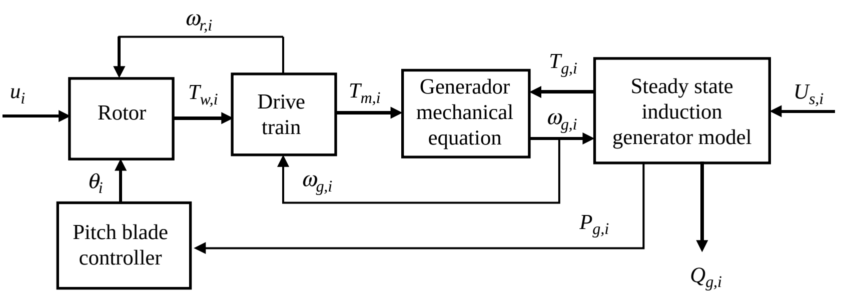
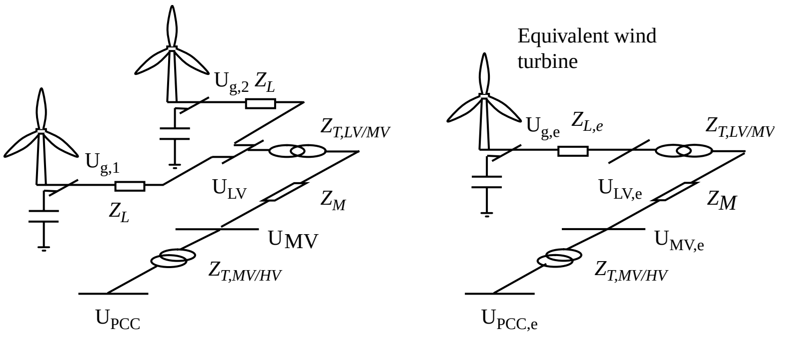


Fig. 10. Simplified wind turbine model.



(a) Complete wind farm (b) Equivalent wind farm

Fig. 11. Complete and equivalent wind farms for steady-state simulations.

voltage line at 20 kV (MV) connects it to the wind farm substation. The wind farm substation present a transformer 66/20 kV coupling at 66 kV (HV) the wind farm to grid, as shown in Fig. 11. The numeric parameters of the wind farm under consideration are shown in Table 1.

The behaviour of the complete wind farm has been represented by a detailed model including the modelling of all the wind turbines and wind farm electrical network.

Three equivalent wind farms have been considered, each one represented by a single equivalent wind turbine operating on an equivalent electrical network. The equivalent wind turbines considered in the equivalent wind farms are the following:

(i) Equivalent wind turbine with equivalent wind, denoted as EM1.
(ii) Equivalent wind turbine with simplified wind turbine model for calculating the generator mechanical torque of each individual wind turbine and constant equivalent compensating capacitor of those of the aggregated wind turbines, denoted as EM2.
(iii) Equivalent wind turbine with simplified wind turbine model for calculating the generator mechanical torque and reactive power of each individual wind turbine and variable equivalent compensating capacitor, denoted as EM3.

The electrical network of equivalent wind farms, presented in Fig. 11, is composed of the equivalent LV electric line of aggregated wind turbines, LV/MV transformer, MV electric line and MV/HV transformer.

The equivalent wind farm models have been compared with the detailed wind farm model under different constant winds incident on the aggregated wind turbines, as shown in Fig. 12, where the presented variables are the differences between the values of complete and equivalent wind farms.

Fig. 12a shows the wind farm simulation results considering that the wind turbine 1 operates with constant wind speed of 5 m/s and the wind turbine 2 under different wind speed from 5 to 25 m/s. In Fig. 12b, it has been considered that the wind turbine 1

Table 1
Wind farm numeric parameters

Parameter	Symbol	Value	Unit
Wind turbine			
Base power	S_B	350	kVA
Air density	ρ	1.12	kg/m^3
Rotor diameter	D	30.4	m
Rotor inertia	J_r	5	s
Gearbox ratio	1:N	1:44.38	p.u.
Mechanical coupling stiffness	K_{mc}	100	p.u.
Mechanical coupling damping	D_{mc}	30	p.u.
Generator			
Base power	S_B	350	kVA
Base voltage	U_B	660	V
Stator resistance	R_s	0.00571	p.u.
Stator leakage reactance	$X_{\sigma s}$	0.00690	p.u.
Rotor resistance	R_r	0.00612	p.u.
Rotor leakage reactance	$X_{\sigma r}$	0.18781	p.u.
Magnetizing reactance	X_m	2.78	p.u.
Generator inertia	J_g	1	s
Compensating capacitors			
Reactance	X_c	1.7	p.u.
Wind farm electrical network			
Base power	S_B	0.35	MVA
Base voltage	U_B	660	V
Low voltage line impedance	$Z_{Li,j}$	$0.02+j\cdot 0.008$	p.u.
LV/MV transformers impedance	$Z_{T,LV/MV,i}$	$0.006+j\cdot 0.03$	p.u.
Médium voltage line impedance	Z_{Mi}	$(2+j\cdot 15)\times 10^{-5}$	p.u.
MV/HV transformer impedance	$Z_{T,MV/HV}$	$0.005+j\cdot 0.01$	p.u.
External grid			
Base power	S_B	0.35	MVA
Short circuit power at PCC	S_{cc}	500	MVA
X/R ratio	X/R	20	p.u.

generates the maximum power (wind speed of 16 m/s) and the wind speed incident on the wind turbine 2 varies from 25 to 5 m/s.

The analysis of simulation results allows the application limits of equivalent models to be established according to the differences between the winds of the aggregated turbines. This analysis has been performed by comparing the following parameters: (i) the active and reactive powers of equivalent wind turbines with the aggregation of those powers of aggregated wind turbines; (ii) the generation voltage of equivalent wind turbines with those of aggregated wind turbines; (iii) the active and reactive powers of equivalent wind farms with those of complete wind farm; (iv) the nodes voltage of equivalent wind farms networks with those of complete wind farm.

As a conclusion, the equivalent wind turbine model with equivalent wind (EM1) is valid for aggregated wind turbines with similar winds, differences of wind speed less than 2 m/s for stalled regulated wind turbines, and therefore with similar operational points. The differences between the winds of the aggregated wind turbines lead to differences

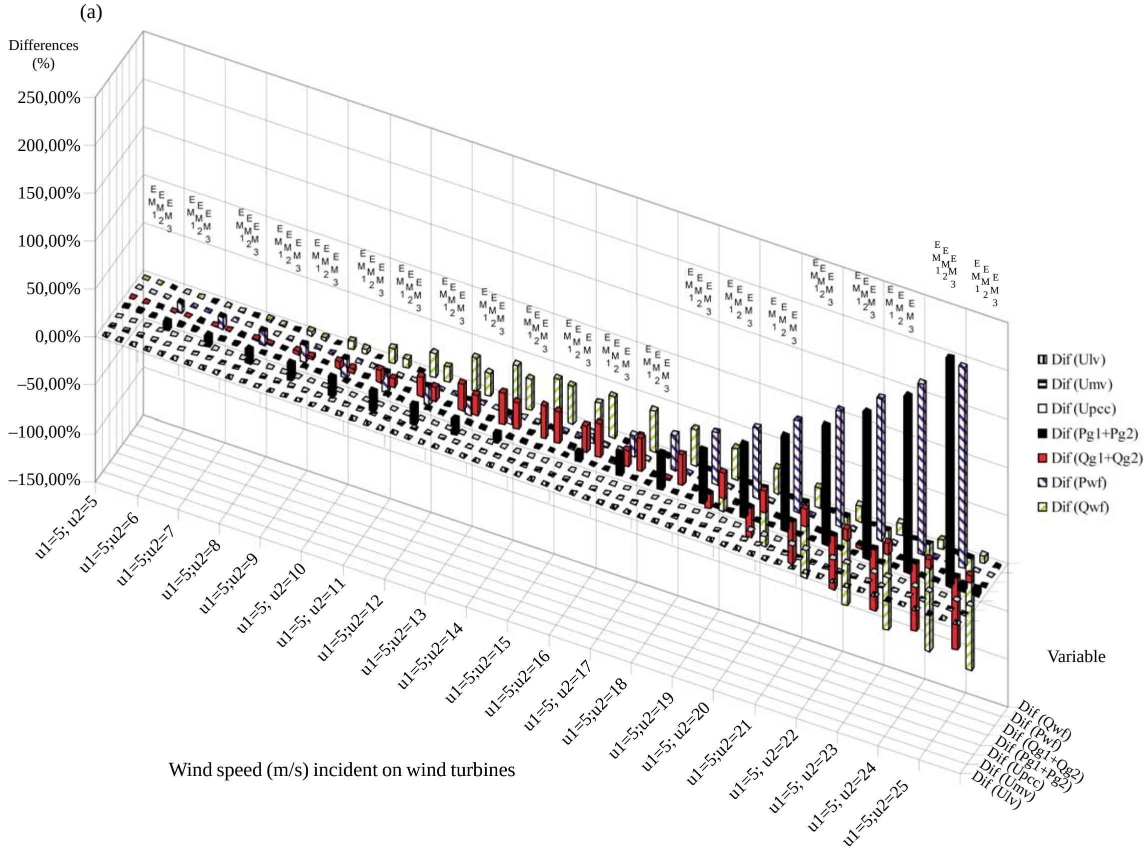


Fig. 12. Comparison between the steady-state response of the complete and equivalent wind farms.

(b)

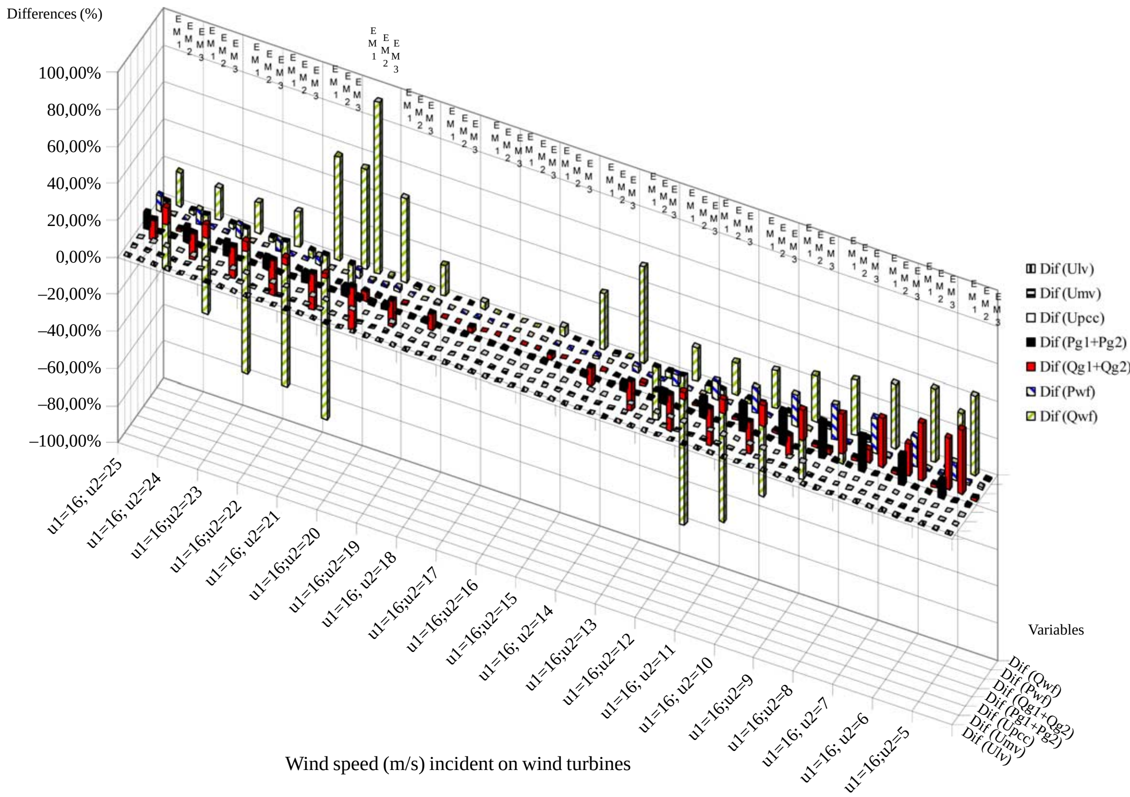


Fig. 12 (*continued*)

between the output power because of the different aerodynamic power and torque of turbines, and therefore the generation voltage and reactive power of aggregated wind turbines are different. Thus the equivalent wind turbine with equivalent wind is not valid for aggregated wind turbines under different winds because of assuming similar operational points in the aggregated wind turbines is false in this case. This model is also valid for pitch regulated wind turbines when the incoming wind is above nominal for all the aggregated wind turbines since the pitch angle controller assures the rated power generation.

The equivalent wind turbines with simplified model of each individual wind turbine, both EM2 and EM3, provide accuracy approximation of active power both of aggregated wind turbines and wind farm under any wind speed. This demonstrates that aggregating the generator mechanical torque of individual wind turbines for wind turbines with almost constant rotational speed enables a good approximation of output power of aggregated wind turbines.

The EM2 model is valid for representing the reactive power both of aggregated wind turbines and wind farm with differences between the winds of the wind turbines less than 4 m/s, and thus resulting reactive power differences less than 5%. The differences over the output power between the aggregated wind turbines lead to differences over the generation voltage and the reactive power. Therefore, the generation voltage of equivalent wind turbine differs from those of individual wind turbines and the reactive power of complete and equivalent wind farms too. This model is also valid for pitch regulated wind turbines when the incoming wind is above nominal for all the aggregated wind turbines.

Finally, the EM3 model results valid for simulating the wind farm response under any wind speed incident on the aggregated wind turbines both stall regulated and pitch regulated, since the simplified wind turbine model approximates the operational points of each individual wind turbine considering the incoming wind speed. Thus, it is demonstrated that aggregating the generator mechanical torque of each individual wind turbine allows a good approximation of the active power of aggregated wind turbines and the variable equivalent compensating capacitor is capable to represent the reactive power of aggregated wind turbines.

These application limits have been checked by dynamic simulations of possible operation conditions in wind farms.

4.2. Dynamic simulations

Dynamic simulations have been carried out in order to evaluate the response of equivalent wind farms under different operation conditions in the wind farm: (i) wind fluctuations in the wind farm; (ii) short circuit at the point common coupling (PCC) of wind farm to grid as grid disturbance.

The wind farm under consideration for evaluating the wind farm models is shown in Fig. 13. This wind farm is composed of six wind turbines of 350 kW, 660 V each, organized into a network of three groups with two wind turbines in each group. Each wind turbine is connected to the wind farm electrical network through a LV electric line at 660 V. Each group is connected to the wind farm substation by a transformer 20/0.66 kV and a MV line at 20 kV. The wind farm substation presents a transformer 66/20 kV

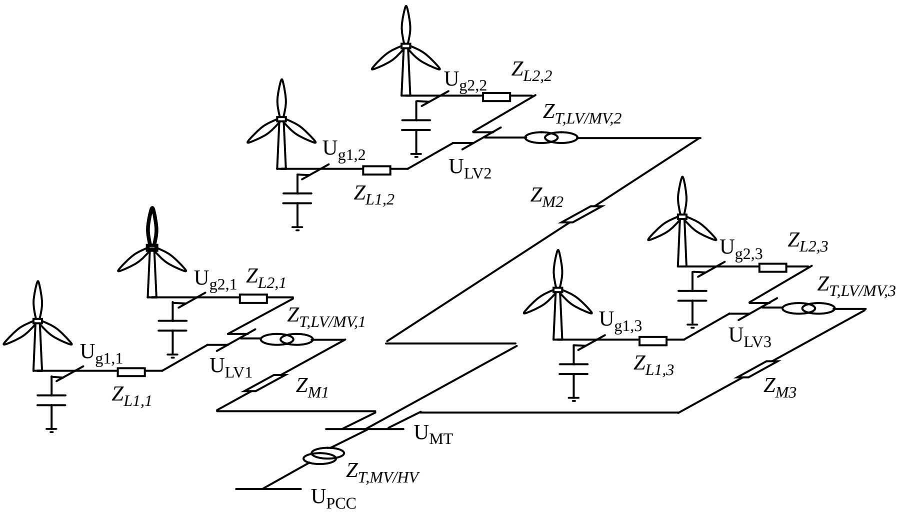


Fig. 13. Complete wind farm for dynamic simulations.

coupling at 66 kV the wind farm to grid in PCC. The numeric parameters of wind farm under consideration are shown in Table 1. Again a detailed model has been used for representing the behaviour of the complete wind farm.

A pair of indexes has been used to identify the wind turbines within the wind farm. The second index from 1 to 3 denotes the number of the group and the first index from 1 to 2 is the number of the wind turbine within the group.

In this case, four equivalent wind farms have been considered, as shown in Fig. 14:

(i) Equivalent wind farm with one equivalent wind turbine experiencing an equivalent wind, denoted as EWF1.
(ii) Equivalent wind farm with three equivalent wind turbines experiencing an equivalent wind (one for each wind turbines group with similar winds), denoted as EWF2.
(iii) Equivalent wind farm with one equivalent wind turbine including a simplified wind turbine model and constant equivalent compensating capacitors, denoted as EWF3.
(iv) Equivalent wind farm with one equivalent wind turbine including a simplified wind turbine model and variable equivalent compensating capacitors, denoted as EWF4.

The electrical network of the equivalent wind farms with one single equivalent wind turbine has been approximated by means of an equivalent LV electric line (parallel of LV electric lines), an equivalent LV/MV transformer (parallel of LV/MV transformers), an equivalent MV electric line (parallel of MV electric lines) and the MV/HV transformer. The equivalent wind farm with three equivalent wind turbines present identical network to

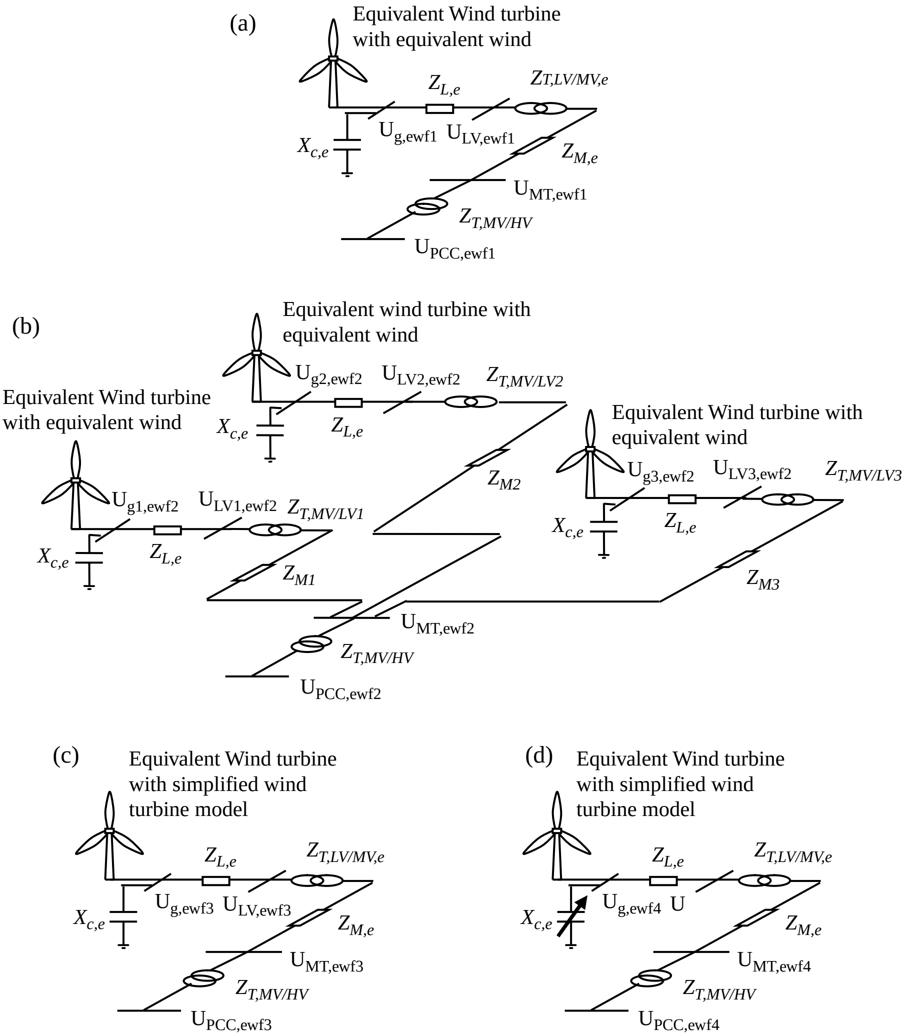


Fig. 14. Equivalent wind farms for dynamic simulations: (a) EWF1 with one equivalent wind turbine experiencing an equivalent wind, (b) EWF2 with three equivalent wind turbines experiencing an equivalent wind (one for each wind turbines group with similar winds), (c) EWF3 with one equivalent wind turbine including simplified wind turbine model and constant equivalent compensating capacitors, (d) EWF4 with one equivalent wind turbine including simplified wind turbine model and variable equivalent compensating capacitors.

complete wind farm excepting the equivalent LV electric line used for each equivalent wind turbine.

Case 1: Wind fluctuations. In this case, the wind farms have been simulated under wind fluctuations in the wind turbines. In the simulated wind farm, it has been assumed that the wind turbines of the same group experience similar winds (variability of winds between the wind turbines less than 2 m/s), and the groups of wind turbines receive different winds,

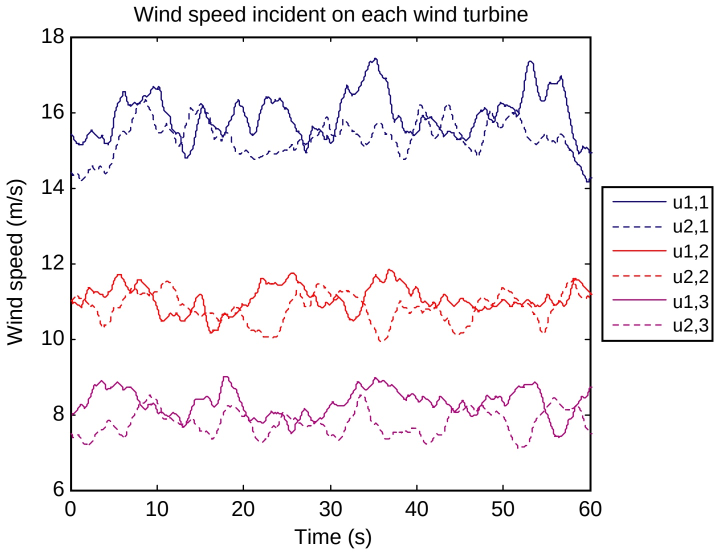


Fig. 15. Wind speed incident on each wind turbine under consideration for dynamic simulations.

as usual for wind farms on topographically simplex terrains. But the differences of winds incident on the groups of wind turbines have been magnified in order to evaluate the equivalent models when aggregating all the wind turbines with important differences between the incoming winds. The wind turbines group 1 receives above nominal winds, while the wind turbines group 2 and 3 present below nominal winds, as shown in Fig. 15.

Two wind farms have been considered, one with stall regulated wind turbines and another with pitch regulated wind turbines, since equivalent models of these wind turbines have been proposed in this work. The simulation results are depicted in Figs. 16–19.

The simulation results show that the equivalent wind farm EWF1 with one single equivalent wind turbine experiencing an equivalent wind is not valid for representing the dynamic response of wind farm because of the differences between the winds incident on the wind turbines. Instead an equivalent wind turbine for each group of wind turbines with similar winds receiving the equivalent wind enables a good approximation of dynamic response of aggregated wind turbines such as it can be derived from the simulation results obtained with the equivalent wind farm EWF2.

The equivalent wind farm EWF3 with one single equivalent wind turbine including a simplified model of each wind turbine and constant equivalent compensating capacitors is capable to approximate the active power of wind farm, because of the approximation of generator mechanical torque obtained by the simplified model, but it fails when representing the wind farm reactive power and the nodes voltage of wind farm electrical network. This problem is solved in the equivalent wind farm EWF4 by using one single equivalent wind turbine that includes a simplified wind turbine model and variable equivalent compensating capacitor, thus enabling an accurate approximation both of active and reactive powers of wind farm.

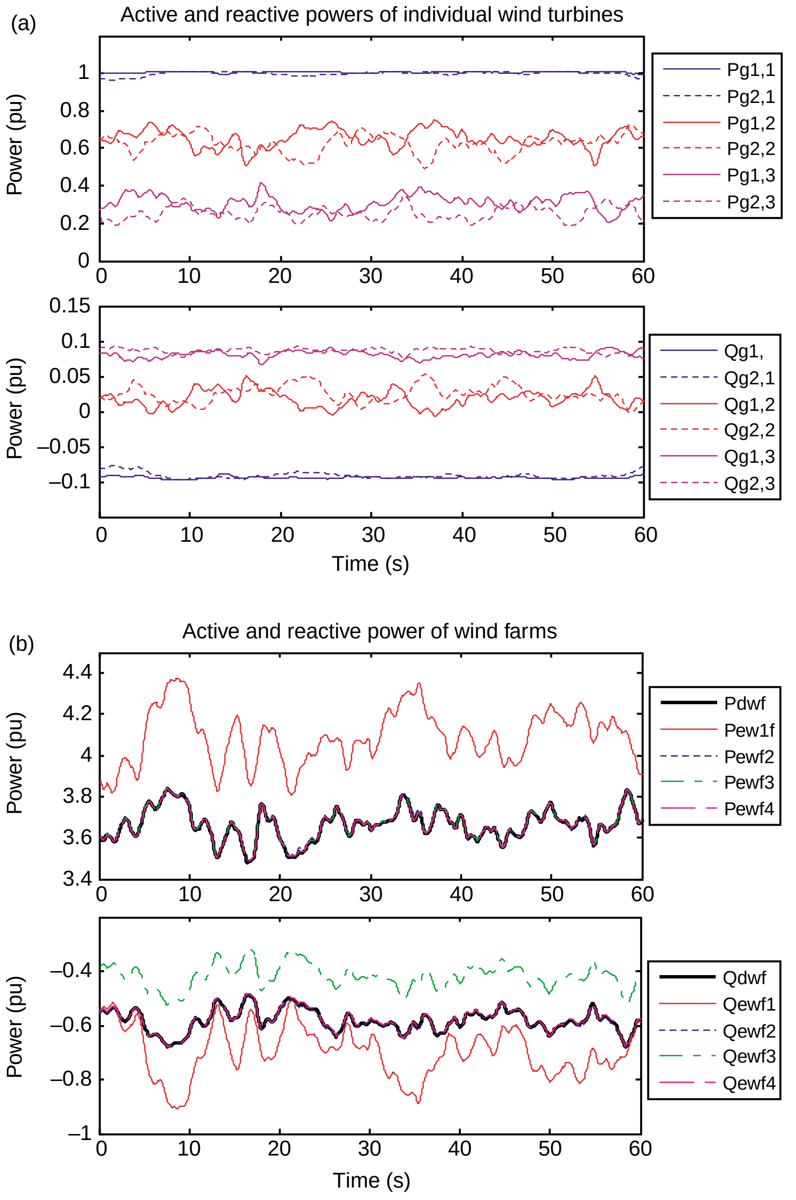


Fig. 16. Active and reactive powers both of wind turbines and wind farm for complete and equivalent wind farms with stall regulated wind turbines under wind fluctuations.

As it was to be expected, these results agree with the conclusions derived from the steady-state simulations. As a conclusion, when comparing the response of equivalent and complete wind farms including groups of wind turbines with similar winds and different winds incident on the groups, the best results are obtained by the equivalent wind farm

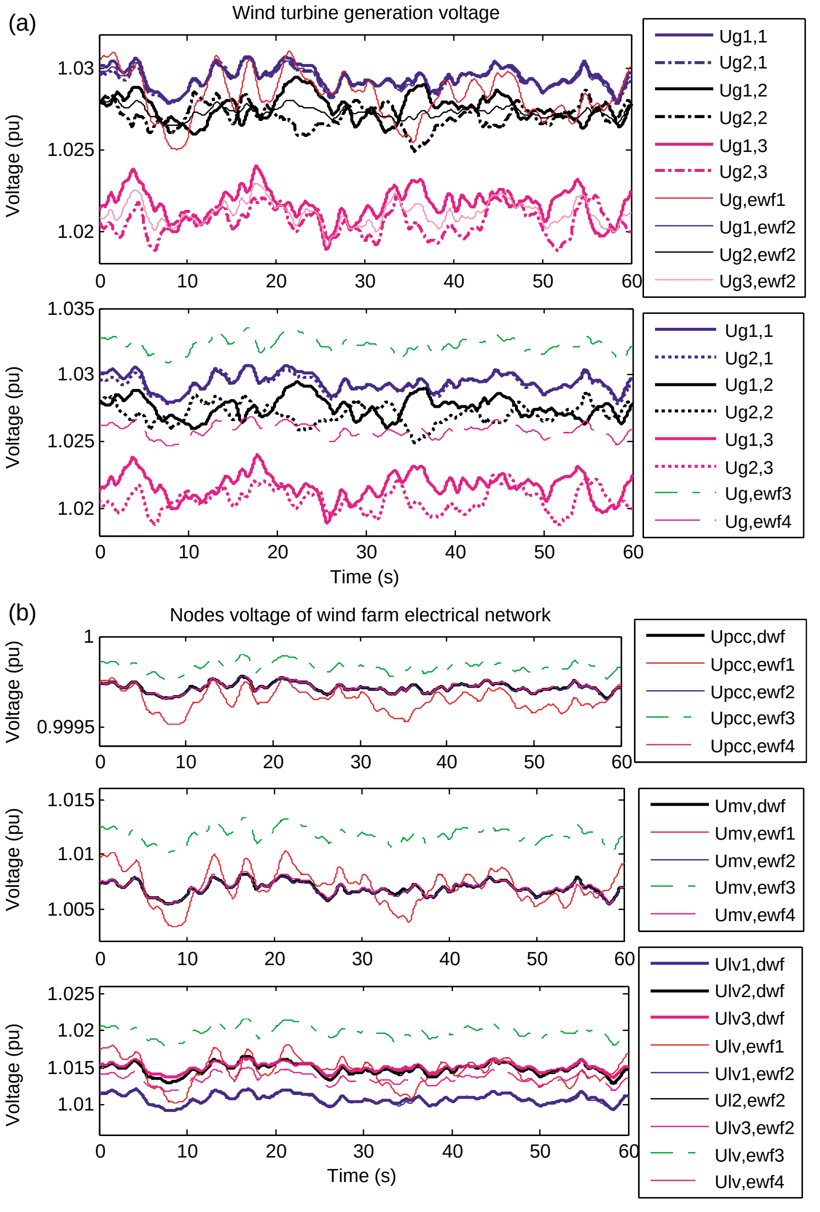


Fig. 17. Nodes voltage of wind farm electrical network for complete and equivalent wind farms with stall regulated wind turbines under wind fluctuations.

EWF2 that uses an equivalent wind turbine with equivalent wind for each group of wind turbines experiencing similar winds and the equivalent wind farm EWF4 with one single equivalent wind turbine including a simplified wind turbine and variable equivalent compensating capacitors for representing all the wind turbines of the wind farm.

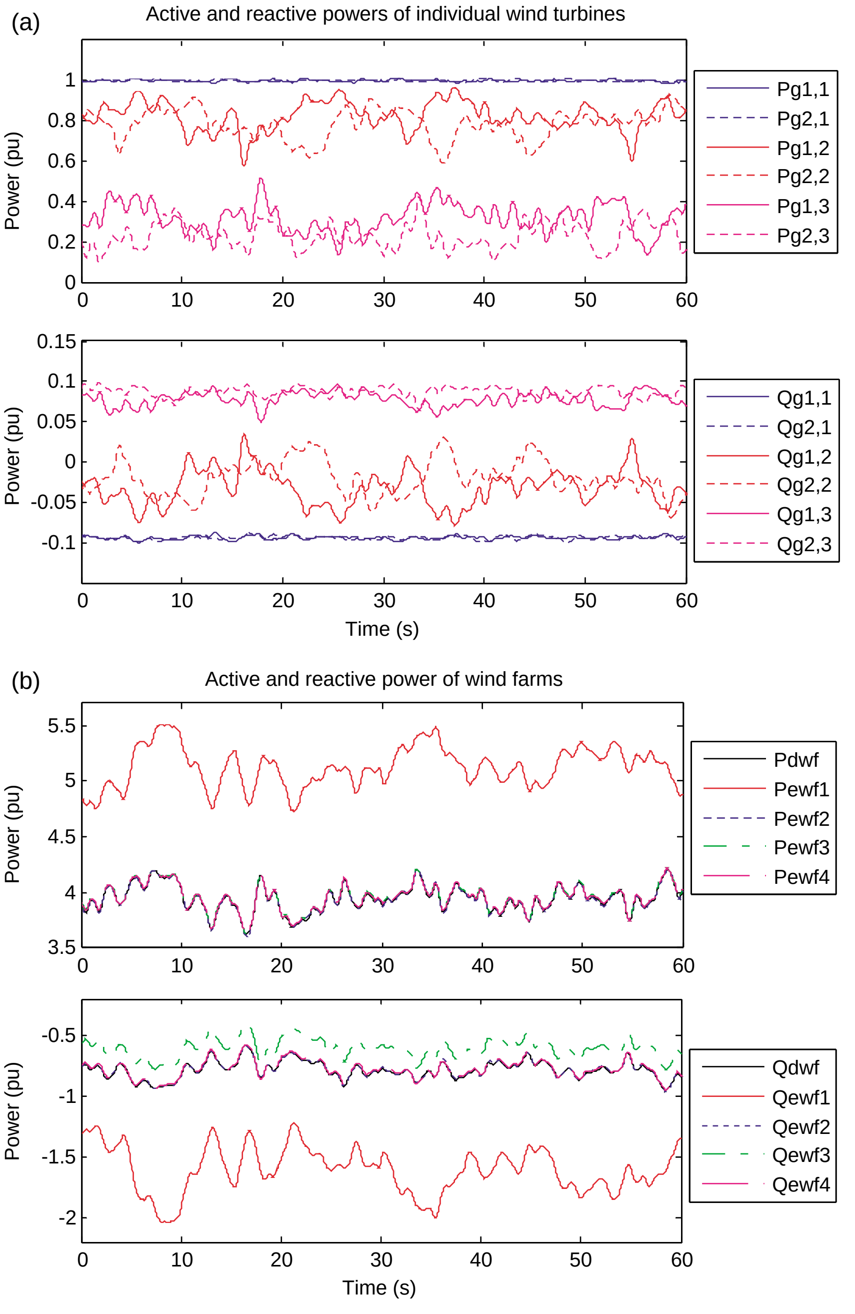


Fig. 18. Active and reactive powers both of wind turbines and wind farm for complete and equivalent wind farms with pitch regulated wind turbines under wind fluctuations.

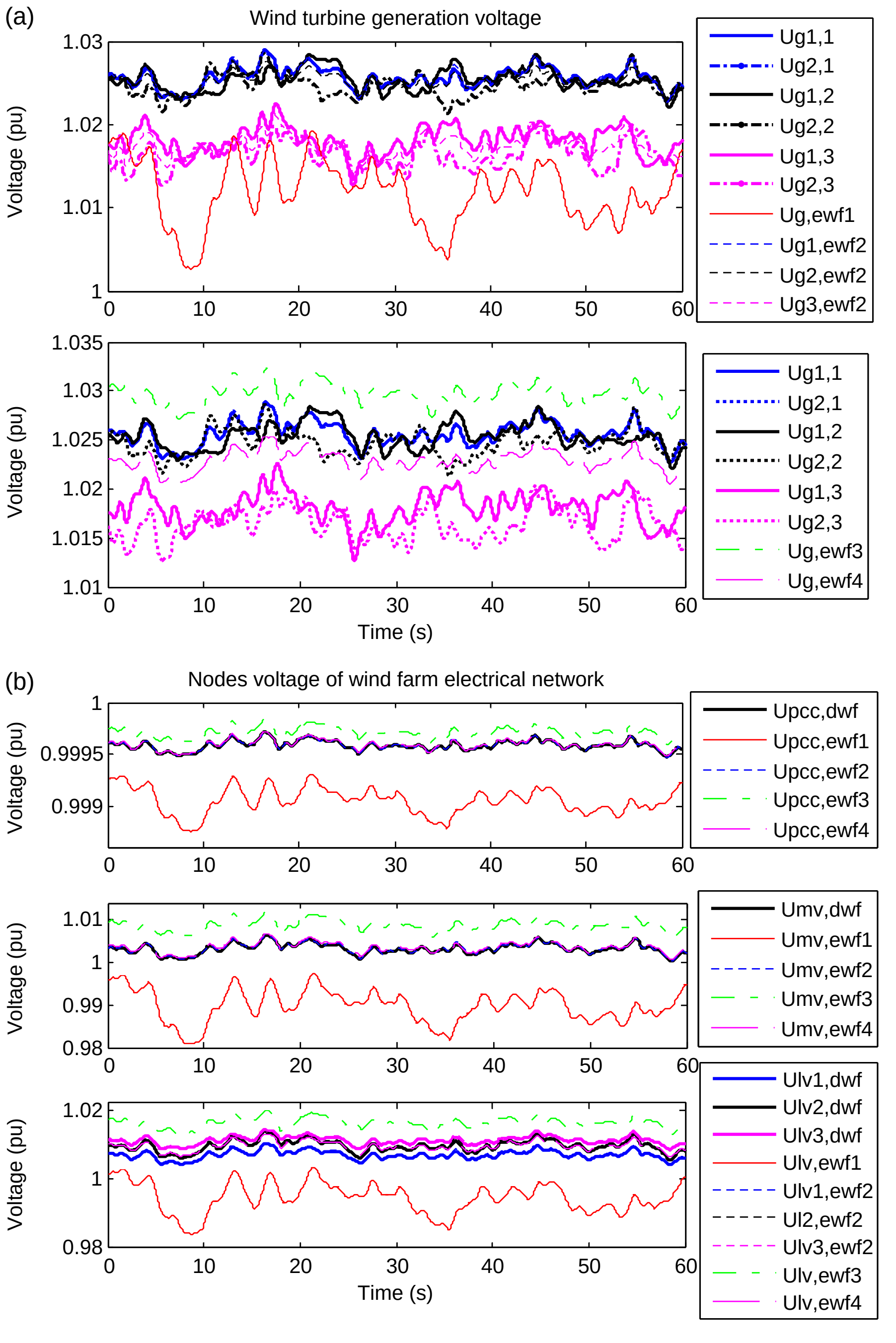


Fig. 19. Nodes voltage of wind farm electrical network for complete and equivalent wind farms with pitch regulated wind turbines under wind fluctuations.

Case 2: Grid disturbance. For evaluating the dynamic response of wind farms under a grid disturbance, the wind farms have been simulated with a short circuit at PCC at $t = 1$ s with a duration of 0.1 s.

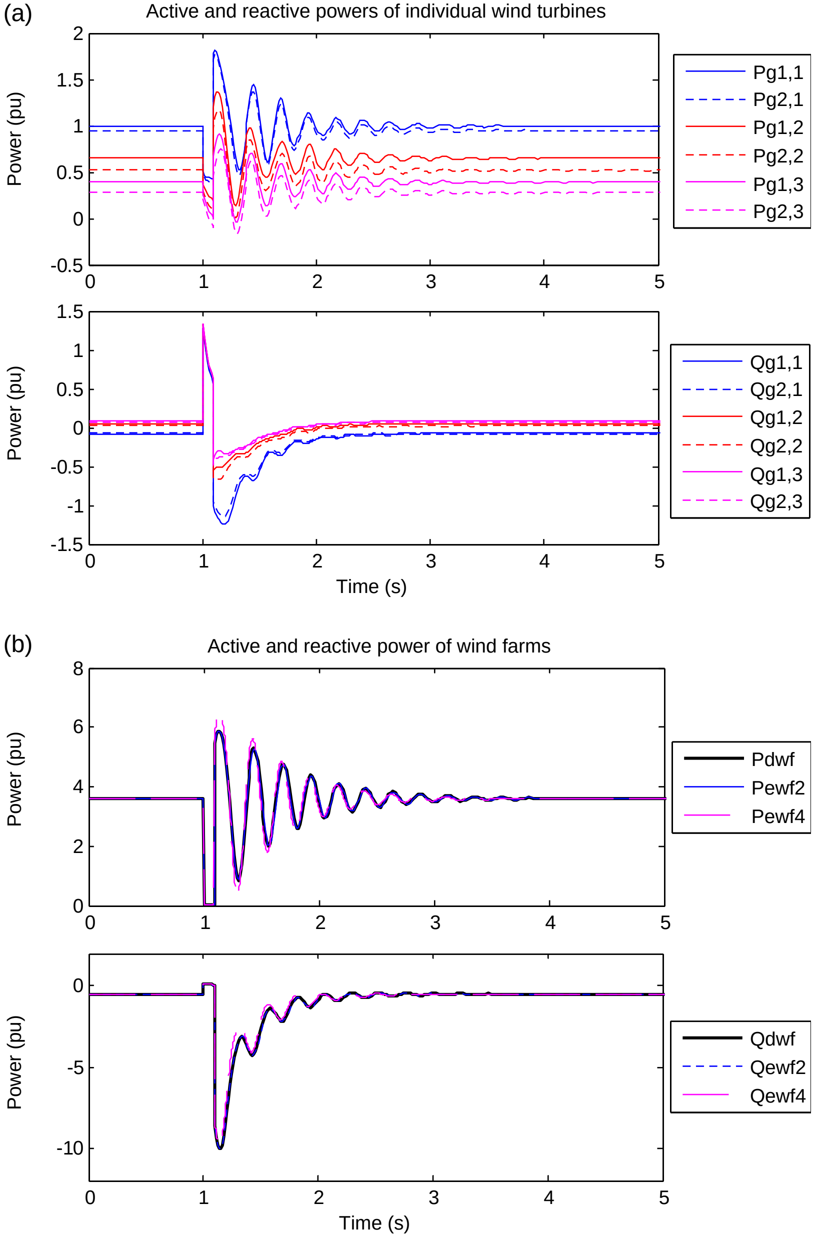


Fig. 20. Simulation results of complete and equivalent wind farms under a short circuit at PCC.

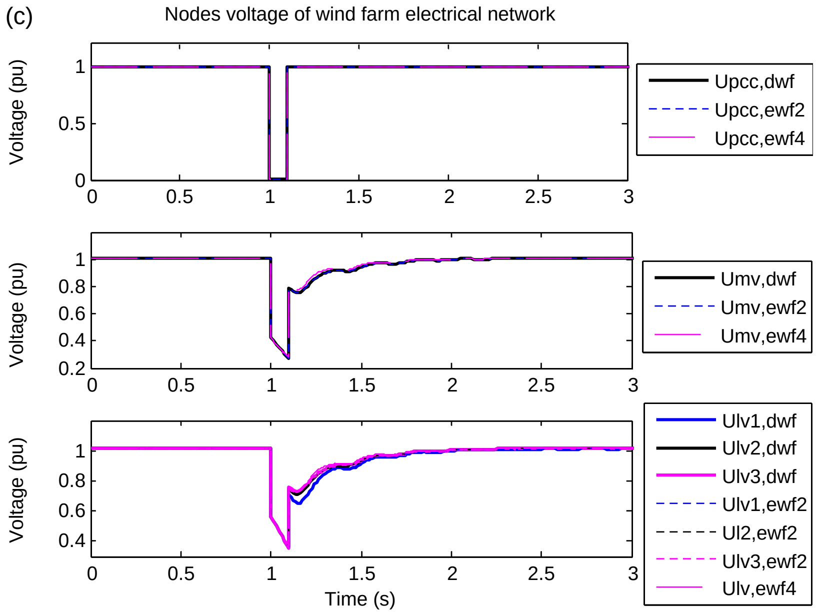


Fig. 20 (*continued*)

As result of the grid disturbances are much faster than wind speed variations, the winds incident on the wind turbines have been assumed constant. Again, it has been assumed that the wind turbines of the same group experience similar winds, and different winds between the groups of wind turbines. Thus the constant wind speeds incident on the wind turbines are the following: group 1 (wind turbine 1,1 with 15 m/s, wind turbine 2,1 with 14 m/s), group 2 (wind turbine 1,2 with 11 m/s, wind turbine 2,2 with 10 m/s) and group 3 (wind turbine 1,3 with 9 m/s, wind turbine 2,3 with 8 m/s). Considering this wind distribution in the simulated wind farm, the equivalent wind farms EWF2 and EWF4 have been used for simulating the dynamic response of wind farms under a grid disturbance.

In this case it has only been simulated the wind farm with stalled regulated wind turbines, since the impact of pitch regulation over wind turbines has a low influence because of the fast dynamic response of grid disturbances as opposed to the one of pitch regulation.

The simulation results are shown in Fig. 20. Fig. 20a depicts the response of the individual wind turbines under the short circuit. When the voltage dip reaches a wind turbine, the generated active power falls, while the mechanical power does not change and therefore the wind turbine accelerates. When the voltage starts to recover, because of the rotational speed has increased, the active power tends to be higher than before the fault. But this also requires higher current, which produces higher voltage drops in the lines and transformers, and therefore the generation voltage of the wind turbines does not recover immediately the value before the fault, but a transient period follows as shown in Fig. 20a.

Again, the comparison of the dynamic response of complete and equivalent wind farms performed by using Fig. 20b and c demonstrates that the equivalent wind turbines used in the simulated equivalent wind farms enable an accurate approximation of the response of complete wind farm under a grid disturbance.

5. Conclusion

In this paper, two new equivalent models of wind farms with fixed speed wind turbines have been presented for representing the dynamic behavior of wind farms on power systems, both for wind fluctuations and grid disturbances, with an important reduction of model order and computation time. The equivalent wind farms here proposed are based on aggregating wind turbines into an equivalent wind turbine that operate on an equivalent wind farm electrical network.

As it has been demonstrated in this work by comparing the simulation results of complete and equivalent wind farms, an equivalent wind turbine experiencing an equivalent wind is capable to represent the behavior of aggregated wind turbines with similar winds. This way to reduce the wind farm model can be used in wind farms placed on topographically simple terrains for aggregating wind turbines groups with similar winds. Therefore, this equivalent wind farm would have so many equivalent wind turbines as wind turbines groups with similar winds.

For aggregating wind turbines under different incoming winds, it has developed an equivalent wind turbine including an aggregated generation system but with variable equivalent compensating capacitors for a better approximation of reactive power of aggregated wind turbines. Because of the difference over the generation of aggregated wind turbines, the turbines cannot be aggregated and these have been replaced by a simplified wind turbine model for approximating the operational points of each wind turbine according to the incoming wind. This equivalent wind turbine can be used for aggregating wind turbines in wind farms on any terrains because of enabling the aggregation of wind turbines under any incoming winds. Furthermore, this equivalent model allows the representation of a wind farm by a single equivalent wind turbine.

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