Path-Based Distribution Network Modeling: Application to Reconfiguration for Loss Reduction

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***Abstract*—This paper is devoted to efficiently modeling the connectivity of distribution networks, which are structurally meshed but radially operated. A new approach, based on the "path-to-node" concept, is presented, allowing both topological and electrical constraints to be algebraically formulated before the actual radial configuration is determined. In order to illustrate the possibilities of the proposed framework, the problem of network reconfiguration for power loss reduction is considered. Two different optimization algorithms—one resorting to a genetic algorithm and the other solving a conventional mixed-integer linear problem—are fully developed. The validity and effectiveness of the path-based distribution network modeling are demonstrated on different test systems.**

***Index Terms*—Distribution networks, genetic algorithm, loss minimization, network reconfiguration.**

I. Introduction

PLANNING and operation of distribution systems involve a long list of optimization problems, like expansion at minimum cost or network reconfiguration keeping in mind a certain objective function (e.g., feeder and/or substation balancing, loss reduction, restoration with minimum switching, etc.). A key factor when practically implementing those optimization problems refers to the fact that, while distribution networks are structurally meshed, they are radially operated. In fact, most optimization problems are simply aimed at finding the best radial configuration, among a huge number of combinations, for which efficient procedures to check or enforce radiality are needed. Typically, procedures resorting to "branch-to-node" incidence concepts are adopted, in which network connectivity is kept track of by checking the status of each individual branch.

In this paper, a new approach based on "path-to-node" incidence concepts is proposed, allowing both radiality and electrical constraints to be algebraically formulated in a compact and efficient manner, at an stage in which the final radial configuration is still unknown.

For the sake of illustrating the viability and advantages of the proposed scheme, the distribution network reconfiguration problem for power loss reduction is considered. Following [1], several categories of network reconfiguration techniques for power loss reduction can be distinguished.

On the one hand, the safest way to solve the reconfiguration problem consists of trying all possible trees for the system in hand, as done in [2]. Due to the magnitude of the problem and its non linear nature, the use of a blend of optimization and heuristic techniques is justified, particularly if the reconfiguration tool is intended for real-time application. This is the case of [3]–[11]. On the other hand, the combinatorial nature of the problem has led researchers to explore purely heuristic solution techniques [12]–[14]. Finally, a third category comprises those contributions using artificial intelligence, like simulated annealing [15], fuzzy logic [16], genetic algorithms (GAs) [17], or artificial neural networks [18].

In this paper, two methodologies will be explored. The first one performs a linearization of both the objective function and constraints, leading to a mixed-integer linear optimization problem. The resulting model improves the preliminary version presented in [19]. The second technique retains the nonlinear objective function, dealing with the reconfiguration problem by means of a GA. Both methods have been tested on different systems reported in the literature, as well as on synthetic larger systems, reaching in all cases the best known solution.

The paper is organized as follows. The new approach to the connectivity problem, based on node paths, is presented in Section II. Section III formulates the optimal reconfiguration problem in terms of the new variables. In Section IV, further simplifications are introduced so that a mixed-integer linear optimization problem results. Main features of the GA customized for this problem are detailed in Section V. Test results are shown in Section VI, while Section VII summarizes the main contributions of the paper. An Appendix is included providing details on how candidate paths are systematically obtained.

II. Path-Based Connectivity Modeling

A. Motivation

The simplest way of modeling the topology of an electrical network is by means of the branch-to-node incidence matrix A, in which as many rows as connected components are omitted to assure linear independence of the remaining rows. Given a single-component meshed network with $N+1$ buses, a well-known theorem states that a set of N branches is a spanning tree if and only if the respective columns of A constitute a full rank submatrix [20]. Practical application of this theorem requires however that A is transformed into the row echelon form or

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a determinant is evaluated, both of which are time-consuming processes for realistic systems.

For this reason, graph-based algorithms are usually adopted in practice. Given the undirected graph of a single-component network, determining if a candidate set of N branches constitutes a spanning tree reduces to checking if they form a single connected component. Alternatively, instead of checking for radiality a posteriori, straightforward algorithms are available to generate radial subnetworks, either from scratch or by performing branch exchanges on existing radial networks.

To the best of our knowledge, none of the above procedures allow the radial structure of candidate subnetworks to be fully characterized by applying a priori a set of algebraic equality and/or inequality constraints to the original meshed system, which is an important limitation of the branch-to-node conventional approach when standard mathematical programming tools are used for the solution of optimization problems.

This has motivated the development of the alternative path-based approach proposed below.

B. Path-Based Formulation

For simplicity of presentation and notation, it will be assumed in the sequel that a single substation exists.

A trivial but relevant preliminary observation is that, for a meshed network, there are in general several alternative paths connecting a given bus to the substation, whereas in a radial network each bus is connected to the substation by a single unique path. Furthermore, the union of all node paths gives rise to the entire system.

The connectivity of a meshed network, as well as that of its radial subnetworks, can then be represented by means of paths. Let Π_n^i be the set of paths associated to bus i

$$\Pi_n^i = \{\pi_1^i, \pi_2^i, \ldots, \pi_k^i, \ldots, \pi_p^i\}$$

where each path π_k^i is a set of branches connecting bus i to the substation. As noted above, any radial network is characterized by only one of those paths being active for each bus. Therefore, there is a need to represent the status of each path π_k^i, for which the following binary variable is defined:

$$W_k^i = \begin{cases} 1, & \text{if } \pi_k^i \text{ is the active path for bus } i; \\ 0, & \text{otherwise.} \end{cases}$$

A candidate subnetwork is both connected and radial if the following constraints are satisfied:

- Every node has at most one active path, i.e.,

$$\sum_{k \in \Pi_n^i} W_k^i = 1, \qquad \forall \text{ node } i. \tag{1}$$

- If π_k^i is active, then any path contained in π_k^i must be also active, which can be written as follows:

$$W_k^i \leq W_m^l, \qquad \forall\, \pi_m^l \subset \pi_k^i. \tag{2}$$

The example shown in Fig. 1 will be used to illustrate these concepts. Table I presents all possible paths for this system.

It is worth noting that, for computational efficiency, not all of the possible paths shown in Table I should be considered in practice. For example, assuming the branch lengths represented in Fig. 1 are proportional to their resistance, it is clear that paths π_3^A and π_3^B can be discarded, as they involve much more electrical distance than that of alternative paths for nodes A and B, respectively. Hence, for each node, only those paths whose total resistance does not exceed a previously defined threshold times the lowest node path resistance are considered. This significantly reduces the number of relevant candidate paths for realistic networks.

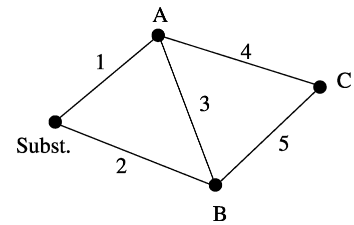

Fig. 1. Example network.

TABLE I
NODE PATHS FOR THE EXAMPLE OF FIG. 1

Node	Paths	Path branches
A	π_1^A	1
	π_2^A	2,3
	π_3^A	2,4,5
B	π_1^B	2
	π_2^B	1,3
	π_3^B	1,4,5
C	π_1^C	1,4
	π_2^C	2,5
	π_3^C	1,3,5
	π_4^C	2,3,4

The inequality constraint (2) is better understood with the help of this example. The reader can easily check that the following inequalities hold[1] (paths π_3^A, π_3^B are discarded)

$$\left.\begin{array}{lllll} W_3^C & \leq & W_2^B & \leq & W_1^A \\ W_1^C & \leq & W_1^A & & \\ W_4^C & \leq & W_2^A & \leq & W_1^B \\ W_2^C & \leq & W_1^B & & \end{array}\right\}.$$

Although the concepts and variables presented above suffice for modeling the network radial structure, in order to handle other branch-related electrical constraints a second set of paths is introduced:

$$\Pi_b^j = \{\text{Set of node paths sharing branch } j\}.$$

Table II shows the set Π_b^j for every branch in the sample system of Fig. 1.

The way sets Π_n^i and Π_b^i are obtained is very important for the computational success of the applications developed below. A graph-based effective procedure is proposed in the Appendix.

III. NETWORK RECONFIGURATION FOR LOSS REDUCTION

It will be assumed that a switch is associated with every line section in the distribution system to be studied.

[1]These inequalities become more evident in Fig. 6 of the Appendix.

TABLE II
SETS Π_b^j FOR THE EXAMPLE OF FIG. 1

Branch j	Π_b^j
1	$\Pi_b^1 = \{\pi_1^A, \pi_2^B, \pi_3^B, \pi_1^C, \pi_3^C\}$
2	$\Pi_b^2 = \{\pi_2^A, \pi_3^A, \pi_1^B, \pi_2^C, \pi_4^C\}$
3	$\Pi_b^3 = \{\pi_2^A, \pi_2^B, \pi_3^C, \pi_4^C\}$
4	$\Pi_b^4 = \{\pi_3^A, \pi_3^B, \pi_1^C, \pi_4^C\}$
5	$\Pi_b^5 = \{\pi_3^A, \pi_3^B, \pi_2^C, \pi_3^C\}$

A. Objective Function

The goal consists of finding the radial configuration which minimizes the system active power losses. Consider the line section model and notation shown in Fig. 2.

Resistive losses at branch j can be obtained from

$$P_j^{loss} = R_j I_j^2 = R_j \frac{(P_j^2 + Q_j^2)}{V_j^2}. \tag{3}$$

Then, the function to be minimized is

$$\min \sum_{j \in B} R_j \left[\frac{P_j^2 + Q_j^2}{V_j^2} \right] \tag{4}$$

where B is the set of N closed branches to be determined.

When solving the above problem, the load flow constraints must be considered, leading to a complex mixed-integer nonlinear problem. Such a problem can be notably simplified by taking into account the following customary simplifications (dc load flow):

1) Voltage magnitudes can be assumed to be 1 p.u. Then, (3) reduces to

$$P_j^{loss} \simeq R_j \left(P_j^2 + Q_j^2\right). \tag{5}$$

2) The active power flow P_j comprises the total active load demanded downstream from node j plus the active losses of the respective branches. This latter component of P_j can be omitted as system losses are much smaller than power loads. The same can be said for the reactive power flow Q_j. This way active/reactive power flows are equal to the sum of active/reactive power loads located downstream from node j, that is,

$$\left.\begin{aligned} P_j &= \sum_{i \ni j} P_{Li} \\ Q_j &= \sum_{i \ni j} Q_{Li} \end{aligned}\right\} \tag{6}$$

where P_{Li} and Q_{Li} are the active and reactive power absorbed by load at node i, and $i \ni j$ represents the set of nodes i located downstream from node j.

Note that application of (6) requires that the final spanning tree (set B of branches) used to feed all loads be known. Therefore, the network connectivity must be incorporated into the objective function. For this purpose, the power flowing through branch j is expressed in terms of the binary variables associated with the involved paths as follows:

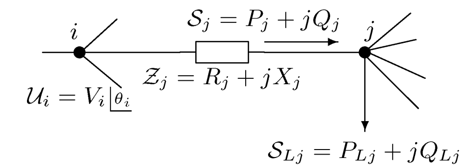

Fig. 2. Line section model and notation adopted.

$$P_j = \sum_{k \in \Pi_b^j} W_k^i P_{Li} \tag{7}$$

$$Q_j = \sum_{k \in \Pi_b^j} W_k^i Q_{Li}. \tag{8}$$

As an example consider branch 3 in the system of Fig. 1, whose active power flow can be expressed as

$$P_3 = W_2^A P_{LA} + W_2^B P_{LB} + W_3^C P_{LC} + W_4^C P_{LC}.$$

If lines 1, 3 and 5 are the active branches in the final configuration, that is, if variables W_1^A, W_2^B and W_3^C are equal to one, then the active power flow through branch 3 is given by $P_{LB} + P_{LC}$.

In view of (7) and (8), the resulting objective function is

$$\min \sum_{j=1}^{b} \left[R_j \left\{ \left(\sum_{k \in \Pi_b^j} W_k^i P_{Li} \right)^2 + \left(\sum_{k \in \Pi_b^j} W_k^i Q_{Li} \right)^2 \right\} \right]. \tag{9}$$

Note that, unlike in (4), the summation extends now to all branches, as network connectivity is incorporated through the binary variables W_k^i.

Power losses estimated in this way will be necessarily smaller than actual losses, because of the two simplifications introduced by (5) and (6).

B. Constraints

When minimizing the objective function (9), both topological and electrical constraints should be enforced. As explained in Section II, the path-based proposed scheme allows radiality constraints to be easily considered by means of (1) and (2).

On the other hand, electrical constraints regarding maximum voltage drop and line ampacity can be modeled as explained below.

The power limit for every line j is given by

$$P_j^2 + Q_j^2 \leq \left(S_j^{max}\right)^2 \tag{10}$$

which, taking into account (7) and (8) yields

$$\left(\sum_{k \in \Pi_b^j} W_k^i P_{Li} \right)^2 + \left(\sum_{k \in \Pi_b^j} W_k^i Q_{Li} \right)^2 \leq \left(S_j^{max}\right)^2. \tag{11}$$

So far as line voltage drop is concerned, a better approximation than the well-known Blondel equation is provided by [12]

$$V_i^2 - V_j^2 \simeq 2(R_j P_j + X_j Q_j) \tag{12}$$

where power losses have been neglected. Then the total quadratic voltage drop through a path π_k^i reaching bus i is approximated by

$$V_s^2 - V_i^2 \simeq 2 \sum_{j \in \pi_k^i} (R_j P_j + X_j Q_j) \tag{13}$$

where V_s denotes the substation voltage. If ΔV_{max} denotes the maximum voltage drop, i.e.,

$$V_s - V_i \leq \Delta V_{max}$$

it can be easily shown that the quadratic voltage drop should be bounded as follows:

$$V_s^2 - V_i^2 \leq \delta_{max}$$

with $\delta_{max} = \Delta V_{max}(2V_s - \Delta V_{max})$. Therefore, keeping in mind (7) and (8), the bus voltage drop limitation for each candidate path π_k^i leading to bus i can be incorporated to the problem as follows:

$$2 \sum_{j \in \pi_k^i} \left[R_j \left(\sum_{m \in \Pi_b^j} W_m^i P_{Li} \right) + X_j \left(\sum_{m \in \Pi_b^j} W_m^i Q_{Li} \right) \right] \leq \delta_{max} + \left[1 - W_k^i\right] \tag{14}$$

where the term $[1 - W_k^i]$ is added to prevent (14) from being binding for any inactive path corresponding to bus i.

IV. Mixed-Integer Linear Model

Owing to the simplifications introduced in the former section, bus complex voltages are removed from the model, which means that no load flow is required during the solution process. The resulting optimization problem is still quadratic in the binary variables W_k^i (path statuses), which precludes the use of commercial optimization packages. In order to obtain a standard mixed-integer linear model, quadratic branch power flows are approximated by a piecewise linear function, as shown in Fig. 3 for P.

Assuming for instance that three linear intervals are chosen, the active power flow P turns out to be

$$\left.\begin{array}{rcl} P & = & P^{(1)} + P^{(2)} + P^{(3)} \\ \text{subject to} & & \\ 0 & \leq & P^{(1)} \leq \overline{P}^{(1)} \\ 0 & \leq & P^{(2)} \leq \left(\overline{P}^{(2)} - \overline{P}^{(1)}\right) \\ 0 & \leq & P^{(3)} \leq \left(\overline{P}^{(3)} - \overline{P}^{(2)}\right) \end{array}\right\}. \tag{15}$$

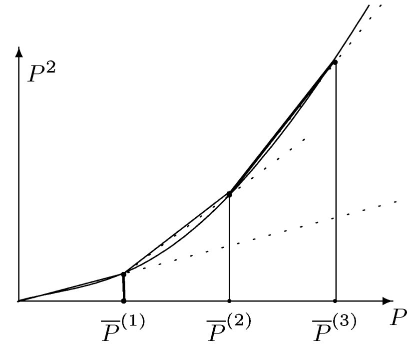

Fig. 3. Piecewise linearization applied to a quadratic function.

The same piecewise linearization can be performed for the reactive power flow. Then, power losses become

$$\begin{aligned} P_j^{loss} \simeq R_j & \left(cp_j^{(1)} P_j^{(1)} + cp_j^{(2)} P_j^{(2)} + cp_j^{(3)} P_j^{(3)} \right) \\ & + R_j \left(cq_j^{(1)} Q_j^{(1)} + cq_j^{(2)} Q_j^{(2)} + cq_j^{(3)} Q_j^{(3)} \right) \end{aligned} \tag{16}$$

where the constants $cp_j^{(t)}$, $cq_j^{(t)}$ ($t = 1,2,3$) are the slope of each segment. Note that different linearization intervals may be in general needed for each branch, both for P and Q. Narrower intervals implies more accurate solutions but also a larger number of variables and constraints. Hence, proper attention should be paid to this issue when practically implementing the proposed algorithm. Reasonable linearization intervals can be obtained if, during the process of determining the sets of node paths (see the Appendix), the maximum and minimum active and reactive power flow for each branch are recorded.

Therefore, the function to be minimized in the reconfiguration problem is

$$\min \sum_{j=1}^{b} \left[R_j \left\{ \sum_{t \in tp} \left(cp_j^{(t)} P_j^{(t)} \right) + \sum_{t \in tq} \left(cq_j^{(t)} Q_j^{(t)} \right) \right\} \right] \tag{17}$$

where tp and tq refer to the number of intervals chosen to linearize the active and reactive power flows, respectively. When formulated as in (17), the objective function must be accompanied by constraints (7) and (8), rewritten in terms of the constituent power flows

$$\begin{aligned} \sum_{t \in tp} P_j^{(t)} &= \sum_{k \in \Pi_b^j} W_k^i P_{Li} \\ \sum_{t \in tq} Q_j^{(t)} &= \sum_{k \in \Pi_b^j} W_k^i Q_{Li}. \end{aligned} \tag{18}$$

It is important to emphasize that, owing to the convexity of power losses, the inequalities $0 \leq cp^{(t-1)} \leq cp^{(t)}$, and $0 \leq cq^{(t-1)} \leq cq^{(t)}$ hold. Therefore, during the search for a minimum of (16), $P_j^{(t)}$ remains null until $P_j^{(t-1)}$ reaches its upper limit. This way, unlike in the nonconvex case, there is no need for extra integer variables to appear in the linearized formulation.

Note also that the piecewise linearization leads to an overestimation of power losses, which somewhat compensates for the simplifications introduced by (5) and (6).

Application of the same linearization methodology to the quadratic constraint (11) is straightforward, yielding

$$\sum_{t\in tp} cp_j^{(t)} P_j^{(t)} + \sum_{t\in tq} cq_j^{(t)} Q_j^{(t)} \le \left(S_j^{max}\right)^2, \ \forall \text{ branch } j. \quad (19)$$

The resulting mixed-integer linear programming problem consists then of minimizing (17), subject to constraints (1), (2), (14), (18), (19) and the piecewise linearization inequalities

$$\left.\begin{array}{lll} 0 & \le & P_j^{(1)} \le \overline{P}_j^{(1)} \\ 0 & \le & P_j^{(2)} \le \left(\overline{P}_j^{(2)} - \overline{P}_j^{(1)}\right) \\ \vdots & \vdots & \vdots \\ 0 & \le & P_j^{(tp)} \le \left(\overline{P}_j^{(tp)} - \overline{P}_j^{(tp-1)}\right) \end{array}\right\} \forall \text{ branch } j$$

$$\left.\begin{array}{lll} 0 & \le & Q_j^{(1)} \le \overline{Q}_j^{(1)} \\ 0 & \le & Q_j^{(2)} \le \left(\overline{Q}_j^{j(2)} - \overline{Q}_j^{(1)}\right) \\ \vdots & \vdots & \vdots \\ 0 & \le & Q_j^{(tq)} \le \left(\overline{Q}_j^{(tq)} - \overline{Q}_j^{(tq-1)}\right) \end{array}\right\} \forall \text{ branch } j.$$

In order to reduce the problem size and to speed up the solution, some additional features are considered.

- As noted earlier, those paths whose electrical length exceeds a certain threshold times the shortest distance to the substation for that node are discarded.
- If the set of paths Π_b^j associated with branch j comprises a single element π_l^i, then the respective flow P_j is constant and equal to P_{Li}, provided $W_l^i = 1$.
- If the set of paths Π_n^i associated with bus i comprises a single element, π_l^i, then $W_l^i = 1$.

V. GA

A GA aimed at solving the quadratic optimization problem, as formulated in Section III, is presented in this section. The purpose of this development is twofold: a) to prove the viability of a GA built upon the path concepts introduced in this paper; b) to assess the adequacy of the results provided by the approximate mixed-integer linear model proposed above.

Nowadays, GAs are well-known optimization techniques, specially adapted to find globally optimal solutions to nonconvex problems in which integer variables are handled [22]. Their search mechanism is based on the principles of natural selection, which are repeatedly applied to guide the evolution of an initial population. Each individual within this population is coded by means of a binary string which can be regarded as the set of genes constituting the chromosomes. The individuals are selected and combined by means of crossover and mutation, giving rise to new individuals. These individuals enter the population and the best individuals are transferred to the next generation.

A. Genetic String

In this application each individual is a binary array with as many bits as candidate paths. Active paths are represented by setting to 1 the respective bit. For convenience, bits are grouped into blocks, each one corresponding to a node, which means that feasible individuals are characterized by a single bit per block being activated.

Turning again to our illustrative example, the structure of an individual will be

Node A		Node B		Node C			
W_1^A	W_2^A	W_1^B	W_2^B	W_1^C	W_2^C	W_3^C	W_4^C

where paths π_3^A and π_3^B have been discarded.

The initial population composed of radial networks is generated in a quasirandom manner.

B. Evaluation Function and Electrical Constraints

Unlike radiality constraints (1) and (2), which are checked each time a new individual is generated, the electrical constraints (11) and (14) are progressively enforced by including them as penalty terms in the fitness function, as follows:

$$P = (1 + A + B)P^{loss}$$

where A is the ratio of the number of overloaded branches to the number of active branches, B is the ratio of the number of buses with excessive voltage drop to the total number of buses, and P^{loss} represents the objective function (9).

C. Genetic Operation

Based on the current population, new individuals are obtained by applying crossover and mutation operators, as described elsewhere [22].

It is important to realize that, when implementing any GA-based procedure, certain parameters, like population size, crossover and mutation rates, etc., have to be properly tuned for the best performance of the optimization process.

VI. Test Results

Both the mixed-integer linear programming and the GA schemes have been tested on four different systems ranging from 32 to 205 buses. The two smallest networks have been thoroughly tested in several previous works [8], [12], [14]. The two largest systems are built by duplicating and triplicating, respectively, the well-known 69-bus system, whose one-line diagram is shown in Fig. 4 (the slack bus is omitted in the replication process). In order to increase the number of loops, extra tie-lines are added to the two synthetic systems, as illustrated in Fig. 5. The parameters for these extra lines are provided in Table III. Letters A, B and C are added to the original bus numbering within each subnetwork.

A. Mixed-Integer Linear Programming

The mixed-integer linear programming problem formulated above has been solved by resorting to the CPLEX module under the framework provided by GAMS [21]. Note that, unlike former contributions to this problem, which are based on

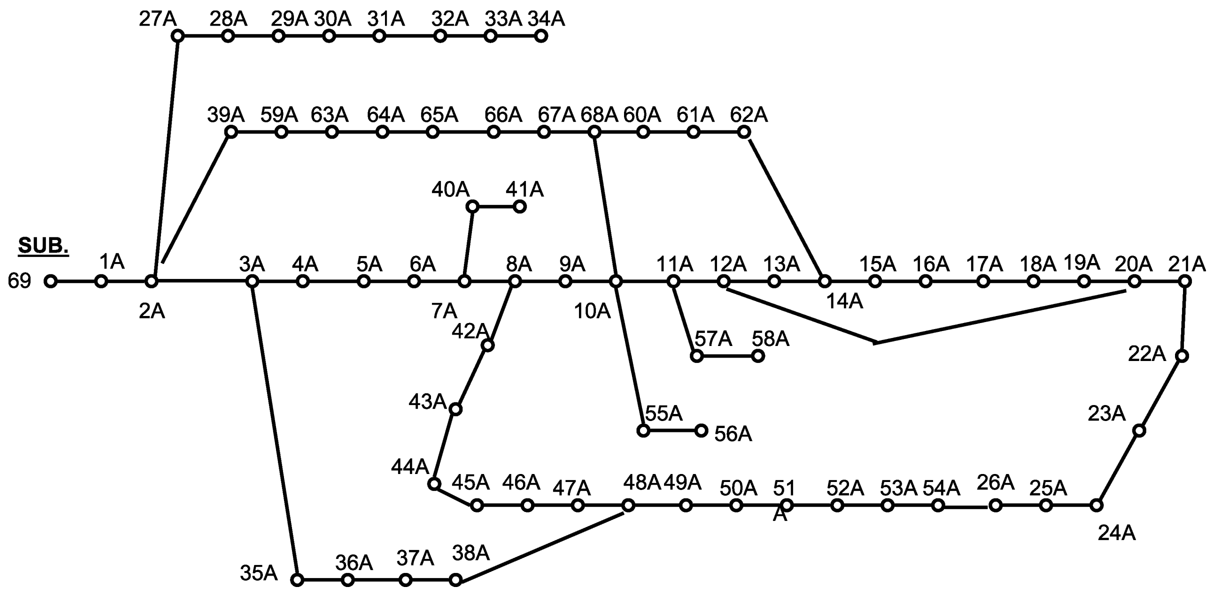

Fig. 4. 69-bus distribution system.

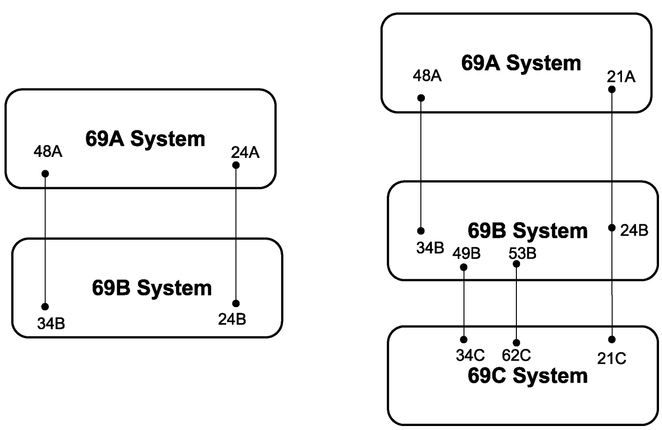

Fig. 5. 137-bus and 205-bus systems.

TABLE III
TIE-LINE PARAMETERS FOR THE 137-BUS AND 205-BUS SYSTEMS

Tie Lines		R	X
Bus From	Bus To	(p.u.)	(p.u.)
48A	34B	0.00000	0.00001
24A	24B	0.00000	0.00001
21A	24B	0.00000	0.00001
49B	34C	0.00000	0.00001
53B	62C	0.00001	0.00002
24B	21C	0.00001	0.00002

hybrid customized algorithms, the results presented below are obtained by directly applying a standard optimization package, without the help of external heuristics. This is only possible when all relevant constraints are algebraically formulated in advance, for which the application of path-based concepts is crucial.

The resulting radial networks are fully defined by as many open sections as existing loops (see Table IV). Table V presents, from left to right, the following data: number of candidate paths considered; number of linearization intervals adopted; exact losses computed by solving the radial load flow, approximate

TABLE IV
RADIAL CONFIGURATIONS PROVIDED BY THE MIXED-INTEGER LINEAR APPROACH

Buses/ Loops	Open Branches (From-To)
32/5	6-7, 31-32, 13-14, 8-9, 24-28
69/5	10A-68A, 13A-14A, 50A-51A, 12A-20A, 47A-48A
137/12	10A-68A, 12A-20A, 13A-14A, 47A-48A, 50A-51A, 24A-24B 10B-68B, 12B-20B, 13B-14B, 47B-48B, 50B-51B, 48A-34B
205/20	10A-68A, 12A-20A, 12A-13A, 47A-48A, 50A-51A, 48A-34B 21A-24B, 23B-24B, 9B-10B, 11B-12B, 17B-18B, 47B-48B 50B-51B, 49B-34C, 12C-13C, 19C-20C, 10C-68C, 20C-21C 47C-48C, 50C-51C

losses provided by the piecewise linearized objective function and resulting relative error (in parenthesis); lowest bus voltage magnitude obtained by the load flow solution (in parenthesis the bus where this happens), lowest bus voltage magnitude approximately obtained by applying (13) followed by the resulting relative error (in parenthesis); total solution time including the process of reading from/writing to files (AMD XP 2200+, 512 MB RAM).

In spite of the objective function underestimating the exact losses by about 4%, the best solution reported up to date for the 32-bus and 69-bus networks is obtained, indicating that the piecewise linear approximation is acceptable for this purpose.

Note that both the number of branch linearization intervals and the number of candidate paths grow with the system size. In these experiments, those paths whose total length exceeds three times that of the shortest path for each node have been discarded. In order to assess whether useful candidate paths may have been inadvertently lost, the 32-bus system is tested by progressively increasing the length ratio threshold. As shown in Table VI, the number of candidate paths reaches a maximum of 623 for this network, but the solution obtained (not shown) is always the same, confirming that it is wasteful to consider longer candidate paths when searching for optimal radial configurations.

As computing technology is continuously evolving, a comparison with former approaches in terms of reported

TABLE V
RELEVANT DATA AND COMPARISON OF RESULTS PROVIDED BY THE MIXED-INTEGER LINEAR APPROACH

No. Buses	No. Paths	No. Linear Intervals	Exact losses (kW)	Approx. losses (kW)	Exact V_{min}	Approx. V_{min}	Solut. time(s)
32	206	4	139.54	135.50 (2.9%)	0.9378 (31)	0.9393 (0.15%)	0.8
69	255	5	30.09	28.60 (4.9%)	0.9452 (50A)	0.9466 (0.15%)	2.4
137	843	7	60.16	57.74 (4.0%)	0.9451 (50A,50B)	0.9466 (0.16%)	19.2
205	1977	8	87.07	83.94 (3.59%)	0.9451 (50C)	0.9436 (0.16%)	134

TABLE VI
NUMBER OF PATHS ACCORDING TO THE LENGTH RATIO THRESHOLD FOR THE 32-BUS SYSTEM

Length Ratio Thresh.	No. Paths	Length Ratio Thresh.	No. Paths
3	206	48	612
6	352	96	623
12	490	192	623
24	553		

TABLE VII
RESULTS OBTAINED BY REDUCING THE MAXIMUM VOLTAGE DROP ALLOWED

Maximum Voltage Drop	Exact Power Losses	Exact V_{min}	Approx. V_{min}
0.1	139.54	0.9378	0.9393
0.06	139.97	0.9413	0.9425
0.056	No feasible solution		

TABLE VIII
RELEVANT DATA AND RESULTS FOR THE GA-BASED PROCEDURE

Num. Buses	Num. Crosses	Population Size	Num. Generations	Losses (kW)
32	10	15	47	139.54
69	25	50	13	30.09
137	50	75	30	60.174
205	50	75	50	90.25

computation times would be meaningless. It is important to realize, however, that the number of paths considered is always much smaller than the number of possible trees. For instance, the 32-bus system comprises over 50 000 different trees [2], whereas the path-based approach considers only about 200 paths. This means that any numerical, heuristic or brute-force approach based on paths will be computationally much cheaper than the counterpart based on trees, while the capability to reach the optimal solution is kept intact.

Another important issue refers to whether the voltage drop estimation error (about 0.15% for the tested cases) affects the solution. The maximum voltage drop adopted for the results presented above is $\Delta V_{max} = 0.1$ p.u., which, according to Table V, means that this constraint is never binding. In order to assess the influence of this limit, the 32-bus system is tested by decreasing ΔV_{max} (see Table VII). For $\Delta V_{max} = 0.06$ a new solution with larger losses is found differing from the original one in branch 27–28, rather than 24–28, being opened. No feasible solution is found with $\Delta V_{max} = 0.056$, which is confirmed by the fact that the load flow solution of the fully meshed system leads to larger voltage drops. The bus with lowest voltage magnitude is correctly identified in all cases by using the approximate expression (13).

B. GA

The GA-based procedure provides the same solution for the two smallest networks, as shown in Table VIII, whereas a slightly worse solution is reached for the two largest networks (a value of 0.1 is chosen for the mutation probability in all cases). This confirms again the validity of the piecewise linearization and related simplifications adopted for the purely numerical approach.

While the solution reached by the mixed-integer linear approach is somewhat affected by the number of intervals chosen in the branch power flow linearization process, the GA performance is significantly affected by certain parameters like population size, number of generations, mutation probability, etc. Several runs of the GA have been necessary in order to properly tune such parameters.

The computational effort of the GA is, as expected, much higher than that of the numerical approach.

VII. CONCLUSION

In this paper, a new paradigm for modeling the connectivity of meshed distribution networks is proposed so that its candidate radial subnetworks can be algebraically characterized in an easy and compact manner. The proposed scheme is based on the preliminary identification of "reasonable" alternative paths linking each bus to the substation(s). The network reconfiguration problem for power loss minimization is used to prove the adequacy of the new scheme. Two solution methodologies are tested: a) mixed-integer linear programming, requiring that both the objective function and electrical constraints be previously converted to piecewise linear functions; b) GA-based heuristic procedure. Test results show that the best known solution is reached in all cases where a comparison is possible.

Main advantages of using the path concept are as follows.

- Graph-based and heuristic methods are needed only to identify candidate paths. Once the set of paths is generated, all electrical and topological constraints can be algebraically formulated, the remaining solution procedure being based on standard well-established optimization techniques.
- Non-logic solutions are discarded from the outset, as those paths whose total resistance is too high are not considered. This significantly reduces the computation time.
- The GA can more easily generate an initial population whose individuals fit the radiality constraints.

APPENDIX
DETERMINING CANDIDATE PATHS

Efficiently obtaining the candidate paths π_k^i and the resulting sets Π_n^i and Π_b^j, for each bus i and branch j, respectively, is a key issue for the computational success of the proposed path-based

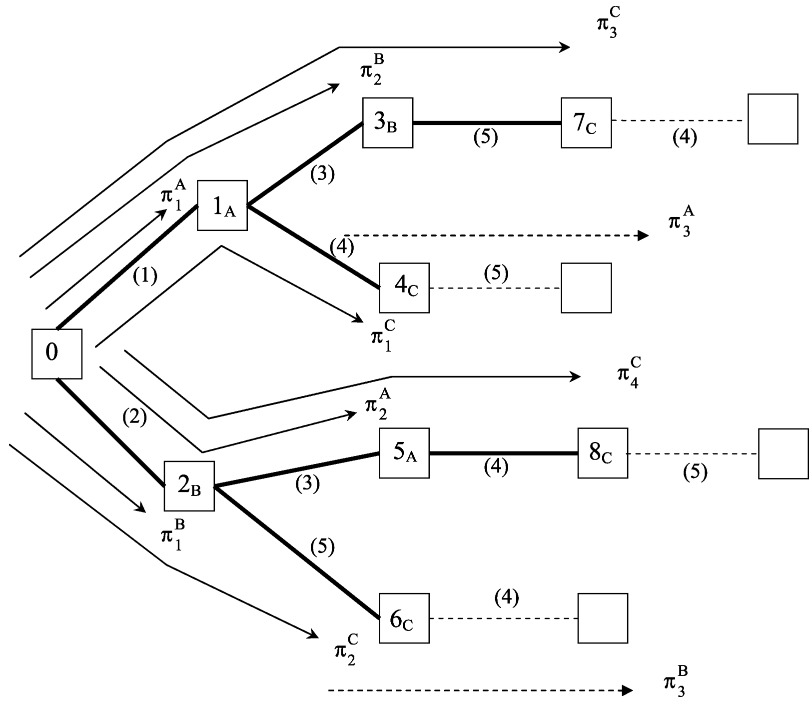

Fig. 6. *Mother tree* for the example of Fig. 1.

methodology. This appendix describes in a concise manner the graph-based procedure adopted in this paper. It is assumed that the meshed network connectivity is conveniently represented by a sparse structure allowing fast access to the set of buses adjacent to a given bus.

The main idea consists of building an auxiliary tree, named *mother tree*, by a breadth-first search, which contains all the feasible paths for the network under study. The system shown in Fig. 1, whose *mother tree* is presented in Fig. 6, will be used once more to better illustrate this concept.

Every node N_L in the *mother tree* corresponds to a possible path for the related bus L. In this case, according to Table I, the four-bus system translates into a *mother tree* with 8 nodes, assuming paths π_3^A and π_3^B are discarded. For example, bus A is associated to nodes 1_A and 5_A in the *mother tree*, corresponding to paths π_1^A and π_2^A (see Table I).

When building the *mother tree* the following rules are taken into account.

- Before a new node N_L is added to the *mother tree*, two conditions are checked,
  - — A node M_L, associated to the same bus L, is not located upstream in the tree. Returning to our tutorial example, a new node, say 9_A, is not appended to node 7_C through branch (4) because bus A already appears upstream in the tree (node 1_A). These dead ends are shown in Fig. 6 by dashed lines.
  - — The impedance of the total path from the substation to the new node N_L does not exceed a threshold times the impedance of the electrically shortest path for bus L. In Fig. 6, paths π_3^A and π_3^B of Table I are not considered for this reason. These cases are represented in Fig. 6 by dashed arrows.
- The *mother tree* is only swept two times, firstly downstream and then upstream. During the downstream swept, both the *mother tree* and associated paths are obtained simultaneously. When the two rules described above preclude the addition of new nodes, the resulting *mother tree* is swept upstream in order to define the inequality constraints among paths, represented by (2), as well as the minimum and maximum power flows through every branch in the system.

In order for the radiality and electrical constraints to be easily expressed in the standard matrix-vector form, sets Π_n^i and Π_b^j are stored as sparse linked lists.

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