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Lie Isomorphisms on H^*-Algebras

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ABSTRACT

We describe the Lie isomorphisms on associative H^*-algebras with zero annihilator.

Key Words: Lie isomorphisms; H^*-algebras; Graded Lie algebras; Lie derivations; Jordan pairs; Associative envelopes.

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1. INTRODUCTION

Over the years, there has been considerable effort made and success in studying the structure of Lie isomorphisms of rings (Banning and Mathieu, 1997; Beidar et al., In press, 2001, Preprint, 1994; Beidar and Chebotar, 2001; Brešar, 1993, 2000; Chebotar, 1996, 1999; Herstein, 1961), and Banach algebras (Berenguer and Villena, 1999; Harpe, 1972; Mathieu, 2000; Miers, 1976). We are interested in investigating the Lie isomorphisms of associative H^*-algebras. Using functional identities one can describe Lie isomorphisms at a rather high level of generality, however, we use entirely different methods to characterize Lie isomorphisms of H^*-algebras. In fact, we note that the introduction of techniques of derivations and graded algebras in the treatment of problems of Lie isomorphisms is perhaps the most interesting novelty in this paper. We recall that an *H^*-algebra A* over $\mathbb{C}$ is a, non-necessarily associative, $\mathbb{C}$-algebra whose underlying vector space is a complex Hilbert space, endowed with a conjugate–linear map $*: A \to A$ $(x \mapsto x^*)$, such that $(x^*)^* = x$, $(xy)^* = y^* x^*$ for any $x, y \in A$ and the following hold

$$(xy|z) = (x|zy^*) = (y|x^*z)$$

for all $x, y, z \in A$. The map $*$ will be called the *involution* of the H^*-algebra. The continuity of the product of A is proved in Cuenca and Rodríguez (1987). We call the H^*-algebra A, *topologically simple* if $A^2 \neq 0$ and A has no nontrivial closed ideals. H^*-algebras were introduced and studied by Ambrose (1945) in the associative case, and have been also considered in the case of the most familiar classes of nonassociative contexts (Cabrera et al., 1992; Cuenca et al., 1990; Cuenca and Rodríguez, 1987; Devapakkian et al., 1975, 1976; Pérez de Guzmán, 1983; Schue, 1960) and even in the general nonassociative contexts (Cuenca and Rodríguez, 1985, 1987). A Lie isomorphism from an associative H^*-algebra A onto an associative H^*-algebra B is a linear bijective mapping f from A onto B such that $f([x, y]) = [f(x), f(y)]$ for all $x, y \in A$. Here and subsequently, the bracket denotes the Lie product, $[x, y] = xy - yx$ on A and B. In Cuenca and Rodríguez (1987) it is proved that any H^*-algebra A with continuous involution splits into the orthogonal direct sum $A = Ann(A) \perp \overline{\mathscr{L}(A^2)}$, where $Ann(A) := \{x \in A : xA = Ax = 0\}$ is the *Annihilator* of A, and $\mathscr{L}(A^2)$ is the closure of the vector span of A^2, which turns out to be an H^*-algebra with zero annihilator. Moreover, each H^*-algebra A with zero annihilator satisfies $A = \overline{\perp I_\alpha}$ where $\{I_\alpha\}_\alpha$ denotes the family of minimal closed ideals of A, each of them being a topologically

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simple H^*-algebra. We also recall that any isomorphism on arbitrary H^*-algebras with zero annihilator is continuous (Cuenca and Rodríguez, 1985). Let H be a complex Hilbert space and let $\mathscr{HS}(H)$ be the algebra of all Hilbert–Schmidt operators on H. If $\{\phi_i\}_{i\in\mathscr{A}}$ is a complete orthonormal system of H and $f, g \in \mathscr{HS}(H)$, then the sum $\sum(f(\phi_i)|g(\phi_i))$ is independent of the choice of $\{\phi_i\}_{i\in\mathscr{A}}$. It can be proved that $\mathscr{HS}(H)$ becomes an associative H^*-algebra under the inner product

$$(f|g) = \sum (f(\phi_i)|g(\phi_i)).$$

and the involution $f \mapsto f^*$, where f^* is the adjoint operator of f, that is, the unique element in $\mathscr{HS}(H)$ such that $(f(x)|\,y) = (x|f^*(y))$ for all $x, y \in H$. It is proved in Ambrose (1945) that any infinite dimensional topologically simple associative H^*-algebra is $*$-isometrically isomorphic (up to a positive multiple of the inner product) to $\mathscr{HS}(H)$ with H a complex Hilbert space of infinite hilbertian dimension. Our purpose is to prove the following theorems.

Theorem 1.1. *Let A and A' be infinite dimensional topologically simple associative H^*-algebras and let f be a Lie isomorphism from A onto A'. Then f is either an isomorphism or the negative of an anti-isomorphism.*

Theorem 1.2. *Let A and A' be associative H^*-algebras with zero annihilator and let f be a Lie isomorphism from A onto A'. Then there exist ideals P, Q of A and a $\mathbb{C}$-linear bijective mapping $f' : A \to A'$ such that $A = P \perp Q$ and if we denote by $\{I_\alpha\}$ the family of the minimal closed ideals of A then:*

1. *f' is an isomorphism from P onto $f'(P)$.*
2. *f' is the negative of an anti-isomorphism from Q onto $f'(Q)$.*
3. *If I_α is infinite dimensional then $f'|_{I_\alpha} = f|_{I_\alpha}$.*
4. *If I_α is finite dimensional then $\delta_\alpha := f'|_{I_\alpha} - f|_{I_\alpha}$ is a linear mapping from I_α onto the center of A' sending commutators to zero.*

In order to prove Theorem 1.1, we are firstly going to associate Jordan H^*-pairs J_A and $J_{A'}$ to A and A' respectively, via a three-graded Lie H^*-algebras construction. Secondly, we are going to extend f to an associative isomorphism F on certain $\mathbb{Z}_2$-graded associative envelopes of J_A and $J_{A'}$. Finally, the theorem will be finished by arguing on F. Theorem 1.2 will be obtained from Theorem 1.1, some considerations on Lie isomorphisms on finite dimensional simple H^*-algebras and the structure theory of H^*-algebras (Cuenca and Rodríguez, 1987).

2. THE THEOREMS

Let K be a unitary commutative ring. A *three-graded K-algebra A* is a K-algebra which splits into the direct sum $A = A_{-1} \oplus A_0 \oplus A_1$ of nonzero K-submodules satisfying $A_0 A_i + A_i A_0 \subset A_i$ for all $i \in \{-1, 0, 1\}$,

$$A_{-1}A_1 + A_1 A_{-1} \subset A_0$$

and $A_1 A_1 = A_{-1} A_{-1} = 0$. A *three-graded H^*-algebra* is an H^*-algebra which is a three-graded algebra such that A_{-1}, A_0, A_1 are closed orthogonal subspaces satisfying $(A_i)^* = A_{-i}$ for $i \in \{-1, 0, 1\}$. We write $A = A_{-1} \perp A_0 \perp A_1$.

Proposition 2.1. *Let $A = A_{-1} \perp A_0 \perp A_1$ be a three-graded H^*-algebra. Then there exists a self-adjoint derivation D on A with minimal polynomial $x^3 - x$, satisfying $* \circ D = -D \circ *$ and such that $A_i = Ker(D - iId)$, $i \in \{-1, 0, 1\}$.*

Proof. It is easy to prove that $D : A \to A$ defined by

$$D(a_{-1} + a_0 + a_1) = -a_{-1} + a_1$$

for all $a = a_{-1} + a_0 + a_1 \in A$ is a self-adjoint derivation on A satisfying $* \circ D = -D \circ *$, $A_i = Ker(D - iId)$, $i \in \{-1, 0, 1\}$ and $D^3 - D = 0$. As $D \neq 0$ and $D^2 \neq \pm Id$, D has minimal polynomial $x^3 - x$. □

Proposition 2.2. *Let A be an H^*-algebra with zero annihilator, and let D be a self-adjoint derivation on A with minimal polynomial $x^3 - x$ and satisfying $* \circ D = -D \circ *$. Then $A = A_{-1} \perp A_0 \perp A_1$ with $A_i = Ker(D - iId)$, $i \in \{-1, 0, 1\}$, is a three-graded H^*-algebra.*

Proof. The proof follows from the primary decomposition of D, the conditions $* \circ D = -D \circ *$ and D is self-adjoint and the fact that D is continuous by Villena (1994). □

The definitions and preliminary results on Jordan H^*-pairs can be found in Calderón and Martín (1998).

Proposition 2.3. *Let $L = L_{-1} \perp L_0 \perp L_1$ be a three graded Lie H^*-algebra. Then $J_L := (L_{-1}, L_1)$ is a Jordan H^*-pair with the quadratic operators*

$$Q^\sigma(x)(y) = [[x, y], x],$$

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for $\sigma \in \{+, -\}$, $x \in L^{\sigma}$ and $y \in L^{-\sigma}$, and with the involution and inner products induced by the ones in L. (Note that the triple products of J_L are $\{x, y, z\}^{\sigma} = [[x, y], z]$).

Proof. It is proved in Neher (1989) that J_L with the above quadratic operators is a Jordan pair. Finally, the involution and inner product of L clearly endow J_L of a Jordan H^*-pair structure. □

If A is an associative H^*-algebra then A^- will denote the Lie H^*-algebra whose underlying vector space, involution and inner product agree with that of A, and whose product is given by $[x, y] = xy - yx$.

Proof of Theorem 1.1. From Sec. 1, there is no loss of generality in writing $A = \mathscr{HS}(H)$ and $A' = \mathscr{HS}(H')$ being H, H' complex Hilbert spaces with infinite hilbertian dimension. Since A^- and $(A')^-$ are topologically simple Lie H^*-algebras (see Cuenca et al., 1990), f is an isomorphism of topologically simple Lie H^*-algebras and therefore f is a continuous map by Cuenca and Rodríguez (1985). Let $\{\phi_i\}_{i \in \mathscr{A}}$ be a complete orthogonal system of H. We can express

$$\{\phi_i\}_{i \in \mathscr{A}} = \{\phi_i\}_{i \in \mathscr{B}} \cup \{\phi_i\}_{i \in \mathscr{C}},$$

being $\mathscr{B} \cup \mathscr{C} = \mathscr{A}$, $\mathscr{B} \cap \mathscr{C} = \emptyset$, $\mathscr{B}$, $\mathscr{C} \neq \emptyset$ and $\mathscr{B}$ a finite set. Consider the mapping $ad(x) : \mathscr{HS}(H) \to \mathscr{HS}(H)$, given by $ad(x)(a) = [x, a]$ being $x : H \to H$ defined by $x(\phi_i) = -\phi_i$ if $i \in \mathscr{B}$ and $x(\phi_i) = 0$ if $i \in \mathscr{C}$. It is easy to prove that $ad(x)$ is a self-adjoint derivation on $\mathscr{HS}(H)$ with minimal polynomial $x^3 - x$ and satisfying

$$* \circ ad(x) = -ad(x) \circ *.$$

By Proposition 2.2, $A = A_{-1} \perp A_0 \perp A_1$ is a three-graded associative H^*-algebra, being $A_{-1} = \mathscr{HS}(H_{\mathscr{C}}, H_{\mathscr{B}})$, $A_0 = \mathscr{HS}(H_{\mathscr{B}}) \oplus \mathscr{HS}(H_{\mathscr{C}})$ and

$$A_1 = \mathscr{HS}(H_{\mathscr{B}}, H_{\mathscr{C}}),$$

where $H_{\mathscr{B}}$, $H_{\mathscr{C}}$ denote the Hilbert subspaces of H associated to $\{\phi_i\}_{i \in \mathscr{B}}$ and $\{\phi_i\}_{i \in \mathscr{C}}$ respectively, and $\mathscr{HS}(H_{\mathscr{C}}, H_{\mathscr{B}})$ is the set of elements $g \in \mathscr{HS}(H)$ such that $g(B) = 0$ and $g(C) \subset B$ (the same applies to

$$\mathscr{HS}(H_{\mathscr{B}}) = \mathscr{HS}(H_{\mathscr{B}}, H_{\mathscr{B}}), \quad \mathscr{HS}(H_{\mathscr{C}}) = \mathscr{HS}(H_{\mathscr{C}}, H_{\mathscr{C}})$$

and $\mathscr{HS}(H_{\mathscr{B}}, H_{\mathscr{C}})$). We have that f provides $(A')^-$ with a Lie H^*-algebra structure by defining the involution for any $a' \in (A')^-$ as $(a')^* := f(a^*)$,

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a being the only element of A such that $f(a) = a'$, and the inner product as $(a' \mid b') := (a \mid b)$, a, b being the only elements of A such that $f(a) = a'$ and $f(b) = b'$. Consider $(A')^-$ with this Lie H^*-algebra structure, then $f: A^- \to (A')^-$ is an isometric $*$-isomorphism of Lie H^*-algebras, this character give us that

$$ad(f(x)) : A' \to A'$$

is also a self-adjoint derivation of A' with minimal polynomial $x^3 - x$ and such that $* \circ ad(f(x)) = -ad(f(x)) \circ *$. By Proposition 2.2 we have that

$$A' = A'_{-1} \perp A'_0 \perp A'_1$$

is an associative H^*-algebra and f is a three-graded isomorphism from A^- onto $(A')^-$, that is, $f(A_i) = A_i'$ for $i \in \{-1, 0, 1\}$. By Proposition 2.3,

$$f : J_A = (A_{-1}, A_1) \to J_{A'} = \left(A'_{-1}, A'_1\right)$$

is also an isometric isomorphism of Jordan H^*-pairs commuting with $*$. As $ad(f(x))$ is a derivation of the associative algebra A', let us note that (A'_{-1}, A'_1) is also an associative H^*-pair.

Since J_A and $J_{A'}$ come from symmetrizing the associative pairs

$$(\mathcal{HS}(H_{\mathcal{C}}, H_{\mathcal{B}}), \mathcal{HS}(H_{\mathcal{B}}, H_{\mathcal{C}}))$$

and

$$(\mathcal{HS}(H'_{\mathcal{C}}, H'_{\mathcal{B}}), \mathcal{HS}(H'_{\mathcal{B}}, H'_{\mathcal{C}}))$$

respectively, and $(\mathcal{HS}(H_{\mathcal{C}}, H_{\mathcal{B}}), \mathcal{HS}(H_{\mathcal{B}}, H_{\mathcal{C}}))$ is a topologically simple associative H^*-pair for the involution and the inner product defined as in $\mathcal{HS}(H)$, see Castellón et al. (1992, Main theorem), we have that J_A and $J_{A'}$ are topologically simple Jordan H^*-pairs which come from symmetrizing associative pairs. Applying D'Amour's result (D'Amour, 1991, Theorem B or Calderón and Martin, 1998, Theorem 1) as in the proof of Calderón and Martin (1998, Theorem 2), f extends to an isomorphism of two-graded associative algebras

$$F : A \oplus A^{op} \to A' \oplus (A')^{op},$$

being $A = \mathcal{HS}(H) \simeq \begin{pmatrix} \mathcal{HS}(H_{\mathcal{B}}) & \mathcal{HS}(H_{\mathcal{C}} H_{\mathcal{B}}) \\ \mathcal{HS}(H_{\mathcal{B}},\ H_{\mathcal{C}}) & \mathcal{HS}(H_{\mathcal{C}}) \end{pmatrix}$ and

$$A' = \mathcal{HS}(H') \simeq \begin{pmatrix} \mathcal{HS}(H'_{\mathcal{B}'}) & \mathcal{HS}\left(H'_{\mathcal{C}'},\ H'_{\mathcal{B}'}\right) \\ \mathcal{HS}\left(H'_{\mathcal{B}'},\ H'_{\mathcal{C}'}\right) & \mathcal{HS}(H'_{\mathcal{C}'}) \end{pmatrix}.$$

We have two possibilities for F, either $F(A \oplus \{0\}) = A \oplus \{0\}$ or

$$F(A \oplus \{0\}) = \{0\} \oplus A^{op}.$$

In the first case $F(a_1 a_2) = F(a_1)F(a_2)$ for any $a_1, a_2 \in A$, being also

$$f(a_i) = F(a_i)$$

for $a_i \in A_{-1} \cup A_1$ because F extends f. We assert that $f(a_0) = F(a_0)$ for all $a_0 \in A_0$. Indeed, as A^- is a topologically simple Lie H^*-algebra, then $(A \oplus A^{op})^- = A^- \oplus (A^{op})^-$ clearly admits a Lie H^*-algebra structure, $(A \oplus A^{op})^-$ having zero annihilator. The same applies to $(A' \oplus (A')^{op})^-$ and then F is continuous (see Sec.1). Since $A_{-1} \perp \overline{[A_{-1}, A_1]} \perp A_1$ is a non-zero closed ideal of A^- and this is a topologically simple Lie H^*-algebra by Cuenca et al. (1990), then

$$A_0 = \overline{[A_{-1}, A_1]}.$$

As f and F are continuous Lie isomorphisms and

$$f(a_i) = F(a_i)$$

for $a_i \in A_{-1} \cup A_1$, we conclude that $f(a_0) = F(a_0)$ for all $a_0 \in A_0$. Therefore $f(a) = F(a)$ for all $a \in A$ what implies $f(xy) = f(x)f(y)$ for any $x, y \in A$. In the second case, that is, $F(a_1\, a_2) = F(a_2)F(a_1)$ for any $a_1, a_2 \in A$, arguing as in the previous case and taking into account that in this case F is a Lie anti-isomorphism, we have that

$$f(a_0) = -F(a_0)$$

for any $a_0 \in A_0$, being $f(a_1) = F(a_1)$ and $f(a_{-1}) = F(a_{-1})$ for any $a_1 \in A_1$ and $a_{-1} \in A_{-1}$. From here it is easy to verify that $f(xy) = -f(y)f(x)$ for all $x, y \in A$. □

Proof of Theorem 1.2. Denote by $\{I_\alpha\}_{\alpha \in \Lambda}$ and by $\{J_\beta\}_{\beta \in \Omega}$ the family of minimal closed ideals of A and A' respectively. Let us consider $J_{\beta_0} \in \{J_\beta\}_{\beta \in \Omega}$.

If J_{β_0} is infinite dimensional, by the classifications of infinite dimensional topologically simple associative (Sec.1) and Lie (Cuenca et al., 1990) H^*-algebras, there exists $I_{\alpha_0} \in \{I_\alpha\}_{\alpha \in \Lambda}$ such that

$$f(I_{\alpha_0}) = J_{\beta_0}.$$

If we denote by $f'_{\alpha 0}$ the restriction of f to I_{α_0}, Theorem 1.1 shows that $f'_{\alpha 0}$ is either an isomorphism or the negative of an anti-isomorphism.

If J_{β_0} is finite dimensional with dim $J_{\beta_0} > 1$, as J_{β_0} is isomorphic to an associative algebra of the type $\mathscr{M}_n(\mathbb{C})$, $n > 1$, then $[J_{\beta_0}, J_{\beta_0}]$, the vector span of $\{[x, y] : x, y \in J_{\beta_0}\}$, is a simple Lie algebra of type A_l. Hence, there exists $I_{\alpha_0} \in \{I_\alpha\}_{\alpha\in\Lambda}$ such that

$$f([I_{\alpha_0}, I_{\alpha_0}]) = [J_{\beta_0},\ J_{\beta_0}].$$

If we denote by f_{α_0} the restriction of f to $[I_{\alpha_0}, I_{\alpha_0}]$, by Jacobson (1962, Theorem 5, p. 283) f_{α_0} extends to either an isomorphism or the negative of an anti-isomorphism $f'_{\alpha_0} : I_{\alpha_0} \to J_{\beta_0}$. If we call

$$\delta_{\alpha_0} := f'_{\alpha_0} - f|_{I_{\alpha_0}} : I_{\alpha_0} \to A',$$

we assert that $\delta_{\alpha_0}(I_{\alpha_0}) \subset Z(A')$ and that $\delta_{\alpha_0}([I_{\alpha_0}, I_{\alpha_0}]) = 0$. Indeed, let us write any element $x \in I_{\alpha_0}$ as $x = c + a$ with $c \in Z(I_{\alpha_0}) \subset Z(A)$ and $a \in [I_{\alpha_0}, I_{\alpha_0}]$, (note that this decomposition is unique). We have that the character either of isomorphism or negative of anti-isomorphism of f'_{α_0} implies

$$f'_{\alpha_0}(c) \in Z(J_{\beta_0}) \subset Z(A').$$

As f is a Lie isomorphism then $f|_{I_{\alpha_0}}(c) \in Z(A')$. Finally, as $f'_{\alpha_0}(a) = f|_{I_{\alpha_0}}(a)$ for any $a \in [I_{\alpha_0}\}, I_{\alpha_0}]$ we conclude that $\delta_{\alpha_0}(I_{\alpha_0}) \subset Z(A')$ and that

$$\delta_{\alpha_0}([I_{\alpha_0}, I_{\alpha_0}]) = 0.$$

Let J_{β_0} be such that dim $J_{\beta_0} = 1$. The facts that A and A' are linearly isomorphic and that we can define the family of linear isomorphisms $\{f_\alpha{}'\}$,

$$f'_\alpha : I_\alpha \to J_\beta$$

among the minimal closed ideals of dimension not 1 imply that there exists a bijection $\sigma : \Lambda_1 \to \Omega_1$, being $\Lambda_1 = \{\alpha \in \Lambda : \dim\ I_\alpha = 1\}$ and

$$\Omega_1 = \{\beta \in \Omega : \dim J_\beta = 1\}.$$

Hence, if we consider the unique $I_{\alpha_0} \in \Lambda_1$ such that $\sigma(\alpha_0) = \beta_0$ then the unique linear isomorphism $f'_{\alpha_0} : I_{\alpha_0} \to J_{\beta_0}$ such that $f'_{\alpha_0}(1) = 1$ turns out to be an isomorphism (of associative algebras) from I_{α_0} onto J_{β_0}.

Let us consider any $J_{\beta_0} \in \{J_\beta\}_{\beta\in\Omega}$ with the unique H^*-structure that makes $f'{}_\alpha$ either an isometric $*$-isomorphism or $-f'_{\alpha_0}$ an isometric $*$-anti-isomorphism. As $A = \overline{\perp_{\alpha\in\Lambda} I_\alpha}$ and $A' = \overline{\perp_{\beta\in\Omega} J_\beta}$, the isometric character of any f'_α, $\alpha \in \Lambda$, allows us to extend $\{f'_\alpha\}_{\alpha\in\Lambda}$ to an isometric linear

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mapping $f' : A \to A'$ such that

$$A = (\overline{\perp_{\alpha \in \Lambda_P} I_\alpha}) \perp (\overline{\perp_{\alpha \in \Lambda_Q} I_\alpha}),$$

with $\Lambda_P \cup \Lambda_Q = \Lambda$, $\Lambda_P \cap \Lambda_Q = \emptyset$, and being f' restricted to $P := \overline{\perp_{\alpha \in \Lambda_P} I_\alpha}$ an isomorphism and f' restricted to $Q := \overline{\perp_{\alpha \in \Lambda_Q} I_\alpha}$ the negative of an anti-isomorphism. It is clear that P, Q and f' satisfy the conditions of Theorem 1.2. □

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