Control Variability and Internal Bore Evolution in the Strait of Gibraltar: A 2-D Two-Layer Model Study

A. Izquierdo[a,e]**, L. Tejedor**[a]**, D. V. Sein**[b,c]**, J. O. Backhaus**[c]**, P. Brandt**[d]**, A. Rubino**[c] **and B. A. Kagan**[a,b]

[a]*Departamento de Física Aplicada, Universidad de Cádiz, Spain*
[b]*Shirshov Institute of Oceanology, Russian Academy of Sciences, St. Petersburg Branch, St. Petersburg, Russia*
[c]*Institut für Meereskunde, Universität Hamburg, Hamburg, Germany*
[d]*Institut für Meereskunde an der Universität Kiel, Kiel, Germany*

A two-dimensional, high-resolution, non-linear, two-layer, free-surface, boundary-fitted co-ordinate, hydrostatic model was applied to study the time–space variability of hydraulic controls and the development of internal bores in the Strait of Gibraltar. The model predicts the occurrence of four averaged (over a tropical month) controls located to the west of the Spartel Sill, at the Spartel and Camarinal Sills and in the Tarifa Narrows. The last of these controls is apparent in the sense that it consists of discrete fragments alternating with subcritical flow regions. The only control which extends over the whole width of the strait is the control at the Camarinal Sill, but it breaks down during neap tide, too. This control exists concurrently with the control in the Tarifa Narrows for short periods, while for much of the tropical month there is either just one or neither of the controls. The model predicts the development of a hydraulic jump and a jump-drop pair near the Camarinal Sill; the appearance of bulges of Mediterranean water to the east and west of the sill; the large-amplitude and small-amplitude internal bores released from the Camarinal Sill, which travel, respectively, eastward and westward, and their transformation due to radial spreading and dissipative effects. Also presented here are the results illustrating the effects of earth's rotation on the internal bores in the Strait of Gibraltar. © 2001 Academic Press

Keywords: hydraulic control; internal bore; time–space variability; Strait of Gibraltar

Introduction

The Strait of Gibraltar is well known for its inverse estuarine circulation in which the low salinity Atlantic water moves towards the Mediterranean Sea in the upper layer, and the high salinity Mediterranean water moves back to the Atlantic in the lower layer. The interface between the Atlantic and Mediterranean waters is in reality spread over several tens of metres. Nevertheless, a simplified two-layer representation of flow is acceptable at least as a useful starting point for studying the flow dynamics in the strait. Three prominent morphometric features of the strait are the deep ($\sim$700 m) Tarifa Narrows in the eastern part, the shallow (a maximum depth of 300 m) Camarinal Sill just west of the contraction, and the broad (in the along-strait direction) Spartel Sill in the western part of the strait (see Figure 1).

Investigations of the strait dynamics that date back to the 17th century (Deacon, 1985) have mainly centred around three problems: two-layer exchange flow, internal bore and non-linear internal wave

evolution, and surface and internal tides. Our focus here is on the time–space variability of hydraulic controls and the development of internal bores in the Strait of Gibraltar with a 2-D high-resolution, non-linear, two-layer, free-surface, boundary-fitted co-ordinate, hydrostatic model as our primary tool. The dynamics and energetics of surface and internal tides and their interaction in the strait will be discussed in another paper.

A detailed analysis of the two-layer exchange flow in the Strait of Gibraltar has been made by Armi and Farmer (1988) and Farmer and Armi (1988). Relying on hydraulic theory for a steady, two-layer flow through a channel with a rectangular cross-section, on the one hand, and the observational data obtained during the Gibraltar Experiment, on the other, they showed that, as applied to the strait, a necessary condition for maximal exchange (the presence of two controls separated by a subcritical flow) was always fulfilled and that, hence, the steady exchange flow in the strait must be maximal. They also stated that the exchange could change in time, therewith remaining maximal. On invoking indirect evidence from a variety
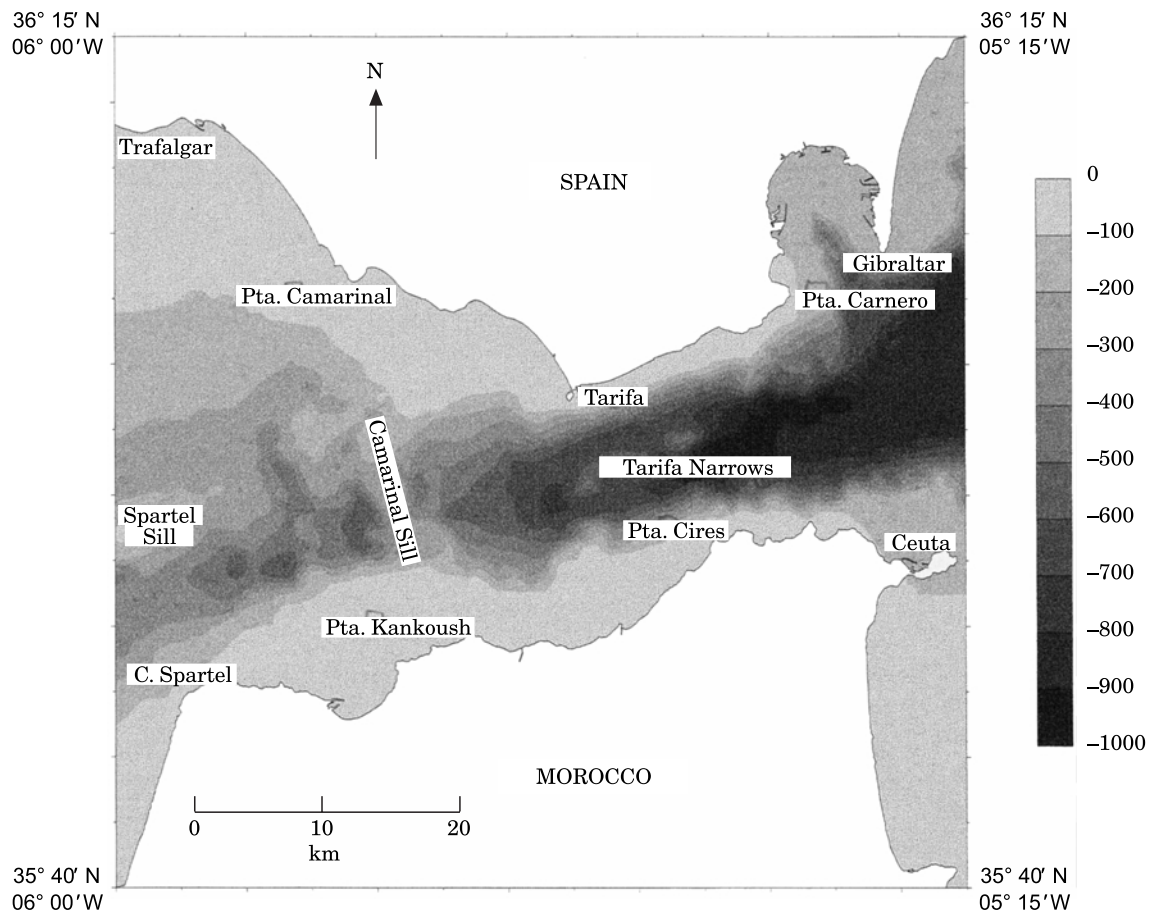
[e]Corresponding author.

FIGURE 1. Map of the Strait of Gibraltar. Superimposed on the bathymetry also shown are the prominent morphometric features of the strait referred to in the text.

of observations, Garret *et al.* (1990) established that the steady exchange in the strait could have either of two possible values, maximal or submaximal, corresponding to supercritical or subcritical flow at the eastern end of the strait, respectively.

The steady hydraulic theory ignores the effects of time dependence, rotation, friction and horizontal two-dimensionality of flow and, against all the odds, provides good agreement between predicted and observed values of the averaged (over a tidal cycle) transport in the strait (Bryden & Kinder, 1991). This may imply that either the above effects are unessential or they are mutually cancelled or, not inconceivably, the observational data are inadequate to quantify exactly the average exchange through the strait. Armi and Farmer (1986) and Farmer and Armi (1986) proposed to account for time dependence parametrically, applying their steady solution at each time of a tidal cycle. However, as argued by Helfrich (1995), this quasi-steady approach fails if the time-scale for long internal waves to propagate through a strait is the

same order as, or longer than, the timescale of tidal or any other barotropic forcing. It is precisely this situation that occurs in the Strait of Gibraltar. Using numerical solutions for both a pure contraction and an offset sill-contraction combination, Helfrich (1995) demonstrated that for a fixed time-periodic barotropic forcing the quasi-steady and unforced steady formulations gave, respectively, upper and lower limiting values of the average exchange. The importance of time dependence to evaluate the average exchange is supported by the experimental estimates of Bryden *et al.* (1994), according to which the unsteady exchange contributes about one-half of the average exchange through the strait.

Another important finding of Helfrich (1995) is that, if account is made for time-dependence and periodic forcing, the average exchange will be dependent not only on geometric conditions at control points, as assumed by steady and quasi-steady two-layer hydraulic theories, but also on details of the geometry of a strait as a whole. But even if the details

of geometry are taken into account, the results from these theories should be interpreted with care: time-dependent, two-layer hydraulic theory, like its steady and quasi-steady prototypes, is based on the 1-D-flow assumption, thereby suggesting that the effects of across-strait non-homogeneity of flow may be neglected. Using a 2-D high-resolution, non-linear, two-layer, boundary-fitted co-ordinate model, Sein et al. (1999) showed that at least for the Strait of Gibraltar this was not the case.

There is a considerable mass of data on internal bores and short-period internal solitary waves in the Strait of Gibraltar (Lacombe et al., 1964; Ziegenbein, 1969, 1970; Kinder, 1984; La Violette & Arnone, 1988; Armi & Farmer, 1988; Farmer & Armi, 1988; Wesson & Gregg, 1994). The data indicate that eastward-travelling, large-amplitude internal bores with ranges from crest to trough of more than 100 m are formed downstream of the Camarinal Sill during tidal inflow. These bores have counterpairs in westward-travelling, small-amplitude internal bores released from the Camarinal Sill at the end of tidal inflow. As predicted by theories of long non-linear internal waves (e.g. Helfrich & Melville, 1990), the bores disintegrate either into rank-ordered sequences of internal solitary waves followed by dispersive wave trains or immediately into dispersive wave trains. This depends on the polarity of initial interfacial disturbances (waves of elevation or depression) which defines the existence condition for internal solitary waves.

In addition to *in situ* measurements, there are many remote observations of surface signatures of internal bores and internal solitary waves in the Strait of Gibraltar obtained by ship-radars (Frassetto, 1964; Ziegenbein, 1969; Cavanie, 1972), shore-based radars (La Violette et al., 1986 Watson & Robinson, 1990) and airborne or satellite synthetic aperture radars, SARs (Alpers & La Violette, 1993; Richez, 1994; Brandt et al., 1996). However, the physics involved in the radar imaging of internal waves are not fully understood (Kropfli et al., 1999; Brandt et al., 1999). This concerns both the physics of internal waves and the physics of radar backscattering from the sea surface.

According to a first-order radar imaging theory, the highest backscattered signal is coincident with the greatest surface strain (surface velocity convergence), see, for example, Alpers (1985) and Brandt et al. (1996). However, recent measurements show examples of radar signatures which cannot be explained by this theory: according to the results of the Coastal Ocean Probing Experiment (Kropfli et al., 1999), in which a detailed comparison was made between *in situ*

observations of internal waves and simultaneous signals from shore-based radars; maximal radar signals (rough sea surface) occur over minimal surface strains, while minimal radar signals (smooth sea surface) tend to be centred over maximal surface strains. That the regions of high/low backscatter are not always coincident with the zones of maximal/minimal surface strain also follows from Thomson (1988), Romeiser and Alpers (1997), and Brandt et al. (1999). Thus, identifying internal waves from the SAR imagery is not without problems.

Nevertheless, several aspects related to the internal wave dynamics can be inferred from radar images showing internal wave signatures indpendently of the assumed image mechanism. For example, Brandt et al. (1996) found a good agreement between measured and simulated positions and distances between the first two internal solitary waves in a wave train in the Strait of Gibraltar at various tidal phases. The basis for this conclusion is a comparison of relative variations of the normalized radar cross-section derived from an analysis of ERS-1 and airborne SAR images and model values of the surface strain. The model in use is a 1-D weakly nonhydrostatic, two-layer model in which the strait is approximated by a channel with a slowly varying (in the along-strait direction) trapezoidal cross-section. Unlike the 1-D hydrostatic, two-layer model of Longo et al. (1992) describing the generation of nonlinear internal tidal waves in an idealized Strait of Gibraltar with a single symmetric sill and no width variations, the above model is capable of simulating the entire sequence of events, namely the generation of interfacial disturbances downstream of the Camarinal Sill, the release of internal bores from the sill, their propagation upstream and ultimately disintegration into internal solitary wave trains.

As known (*e.g.*, Helfrich & Melville, 1990), there are several factors that counteract the nonlinear tendency for internal wave steepening and the formation of discontinuities. These are nonhydrostatic dispersion, forcing, rotation, geometric (lateral) spreading and friction. The model of Brandt et al. (1996) incorporates, in one form or another, all these factors except rotation. Their solution starts from the assumptions of one-dimensional wave propagation and slow variations in both depth and width. These assumptions can lead to erroneous results, especially at the ends of the strait, due to inadequate accounting for variable bottom topography and geometric spreading. The resulting differences between simulated and measured internal wave patterns could be however reduced by a tuning of model parameters (interfacial and bottom drag coefficients). Also, the wide scatter

of ERS-1 and airborne SAR data (whose origin is even difficult to identify) doe not facilitate the anlysis of model predictions.

The purposes of this paper are to: (1) reveal the features of the time-space variability of hydraulic controls in the Strait of Gibraltar using a 2-D high-resolution, nonlinear, two-layer, free-surface, boundary-fitted coordinate, hydrostatic model, and (2) clarify, on the basis of this model, the orderly development, including formation, release and propagation, of internal bores in the strait with an emphasis on the effects of rotation and geometric spreading. The paper is organized as follows. First, the model in use is described, followed by the model results, intended to attain the above ends and a comparison of the model predictions with currently available observational data. A summary and conclusions complete the paper.

The model

The model in use is a 2-D non-linear, two-layer, free-surface, hydrostatic model with the sea-water density taken as being uniform and prescribed in each layer. The governing equations expressed in transport form, read

$$(h_1\mathbf{u}_1)_t + \nabla \cdot h_1\mathbf{u}_1\mathbf{u}_1 + f\mathbf{k} \times h_1\mathbf{u}_1 + gh_1\nabla\zeta_1 = -\tau_1/\rho_1 \quad (1)$$

$$(h_1)_t + \nabla \cdot h_1\mathbf{u}_1 = 0 \quad (2)$$

$$(h_2\mathbf{u}_2)_t + \nabla \cdot h_2\mathbf{u}_2\mathbf{u}_2 + f\mathbf{k} \times h_2\mathbf{u}_2 +$$
$$g(\rho_1/\rho_2)h_2\nabla\zeta_1 + g'h_2\nabla\zeta_2 = (\tau_1 - \tau_2/\rho_2) \quad (3)$$

$$(h_2)_t + \nabla \cdot h_2\mathbf{u}_2 = 0 \quad (4)$$

where h_i, $\mathbf{u}_i$ and ρ_i are, respectively, the layer thickness, depth-averaged velocity and density, with subscripts $i=1$ for the upper layer and $i=2$ for the lower layer; $\zeta_1 = -H + h_1 + h_2$ is the surface elevation; $\zeta_2 = -H + h_2$ is the interface depth; H is the water depth; $\tau_1 = \rho_1 c_1 |\mathbf{u}_1 - \mathbf{u}_2|(\mathbf{u}_1 - \mathbf{u}_2)$ and $\tau_2 = \rho_2 c_2 |\mathbf{u}_2|\mathbf{u}_2$ are the interfacial and bottom stresses, parameterized by a quadratic resistance law with the drag coefficients c_1 and c_2. Also, f denotes the Coriolis parameter; $g' = g(\rho_2 - \rho_1)/\rho_2$ the reduced gravity based on the density difference between the layers; $\mathbf{k}$ a unit vector along the upward z-axis; ∇ the horizontal gradient operator; $(\cdot)$ and $(\times)$, scalar and vector multiplication respectively; and the subscript t indicates differentiation with respect to time.

The model is forced at the open boundaries with radiation-type boundary conditions ensuring that, when short-wavelength disturbances in the fields of variables are generated, they all propagate away from the region of interest. These boundary conditions are given by

$$(h_1\mathbf{u}_1 + h_2\mathbf{u}_2 - H\mathbf{u}) \cdot \mathbf{n} = \pm (gH)^{1/2}$$
$$\left(\zeta_1 - \sum_{n=1}^{N} A_n\cos(\sigma_n t - \varphi_n) \right) \quad (5)$$

for the barotropic tidal mode and

$$(\mathbf{u}_1 - \mathbf{u}_2) \cdot \mathbf{n} = \pm (g'/H')^{1/2}(\zeta_2 - h_1 - (h_2/H)\zeta_1) \quad (6)$$

for the baroclinic tidal mode (mode 1 of internal motion). Here, $\mathbf{u}$ is the averaged velocity (over the whole depth), derived from a solution to the appropriate barotropic problem; A_n and φ_n are the prescribed surface elevation amplitude and phase of the n-th tidal constituent; σ_n is its frequency; N is the number of tidal constituents used to set tidal forcing; $H' = (h_1 h_2)/H$ is an equivalent interface depth; h_1 is the mean interface depth determined, given a certain first guess (see below), by averaging $(\zeta_1 - \zeta_2)$ over a tropical month; $\mathbf{n}$ is a unit vector perpendicular to the open boundary; and the sign used is specified by the direction of wave propagation. It is worth noting that the idea of prescribing a horizontally homogeneous interface depth and a way of its determination were proposed by Brand *et al.* (1996). Here, this method was employed without any changes.

At the coastal boundaries, a condition of now flow normal to the coast is applied. In order to reduce the influence of any inaccuracies in boundary forcing upon the sought-for solution, the model domain is extended to include the eastern part of the Gulf of Cadiz and the western part of the Alboran Sea. In this extended domain, the model equations (1) to (6) are integrated by employing boundary-fitted coordinates. The staggered Arakawa C curvilinear grid in use with a nominal resolution of 1·0 km in the whole model domain and 0·125 km in the Strait of Gibraltar is shown in Figure 2.

The M_2, S_2, K_1 and O_1 surface tidal elevation amplitudes and phases utilized to set the tidal forcing at the open boundary grid points were derived by interpolating the relevant values from a 0·5° gridded version of the FES95.2 global tidal solutions of Le Provost *et al.* (1998). These solutions were obtained by assimilating two years of TOPEX/POSEIDON altimetry data into a finite-element hydrodynamic model. The first guess for the mean interface depth was taken from the solution to the 2-D steady, two-layer exchange flow problem in the Gulf of Cadiz–Strait of Gibraltar–Alboran Sea system given in Sein *et al.* (1999). The bathymetry was deduced from the
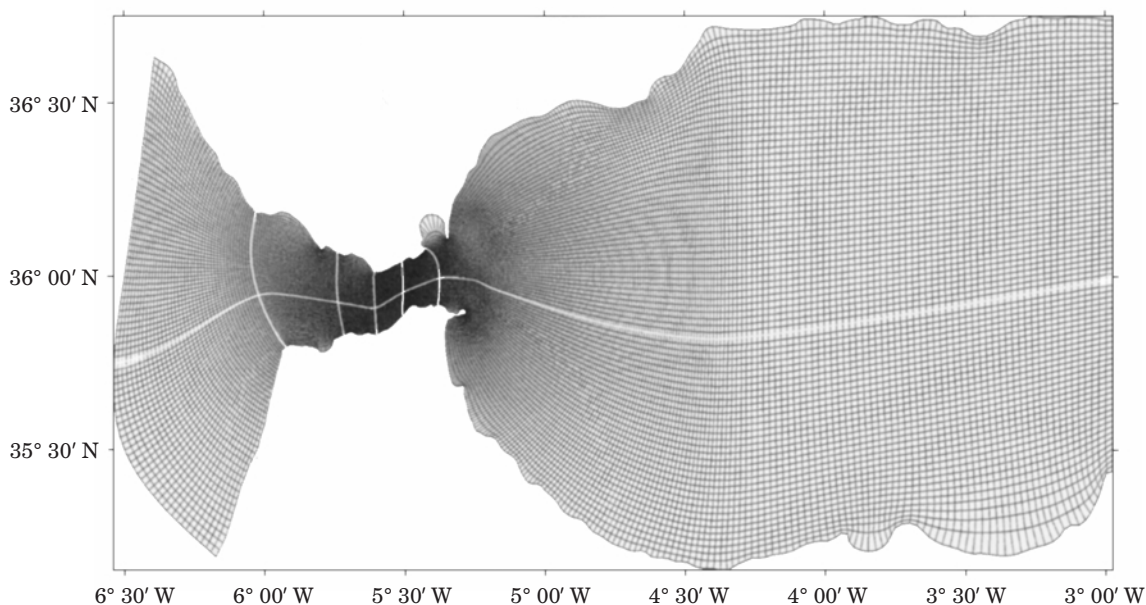
FIGURE 2. Grid map of the model domain. Also shown are the along-strait and cross-sections referred to in Figures 4 and 5.

ETOPO5 database complemented, where required, by the data from the comprehensive chart published by Instituto Geográfico Nacional y SECEG (1988). The values of ρ_1, ρ_2, c_1 and c_2 were specified as 1027 kg m^{-3}, 1029 kg m^{-3}, $0 \cdot 5 \times 10^{-3}$ and 5×10^{-3}, respectively.

For the solution to be smooth, Equations (1) and (3) were supplemented with smoothing terms. These were defined by a horizontal eddy diffusivity operator acting on velocity within the whole model domain except for its boundaries. The horizontal eddy viscosity was kept to a minimum of 10 m^2 s^{-1} to avoid excessively strong smoothing and, at the same time, to suppress short-wavelength disturbances in the field of velocity. The numerical solution was found by employing an alternating-direction technique with a first-order upstream scheme in the horizontal and a semi-implicit Crank-Nicolson scheme in time. A time step restricted by the Courant-Friedrichs-Lewy condition was given a value of 2 s. This time step was used in calculating both the surface and internal tidal modes to avoid averaging of their associated solutions, as would be the case if a time-splitting technique were utilized.

A point that should be mentioned separately is that substantial variations in the interface depth during a tidal cycle can result in vanishing one layer or another, thereby transforming the two-layer representation of flow into a single-layer one. To describe such a situation, the following procedure was implemented: if the thickness of either of the two layers at a certain grid point became less than a critical water depth (say, 1 m), this layer was assumed to be motionless, and the flow was regarded as being uniform and composed of the Mediterranean or Atlantic water only.

The model was run for 30 semidiurnal tidal cycles to achieve a stable time-periodic solution. After establishing this solution, the model run was continued for a 29-day period. The resulting 29-day time-series was then applied to depict the time–space control variability and the internal bore evolution in the Strait of Gibraltar.

Modelling results

In this section we discuss the results obtained from the model and compare them with available observational data. We begin with an analysis of the time–space variability of control locations defined as the locations of transition from subcritical to supercritical exchange flow and back. The internal hydraulic theory of Armi and Farmer (1986) and Farmer and Armi (1986) supposes that such controls, together with the relevant density difference between two water masses and barotropic forcing, completely determine the exchange through a strait.

Control variability

A detailed analysis of the control locations in the Strait of Gibraltar has been provided by Armi and Farmer (1988) and Farmer and Armi (1988) on the basis of

the observations taken in the Gibraltar Experiment. According to these authors, there are four controls in the strait, of which two are permanent and two are episodic. The permanent controls occur to the west of the Spartel Sill and in the central or western part of the Tarifa Narrows, while the episodic controls are sited over the Spartel and Camarinal Sills. As their name shows, the episodic controls exist only at certain stages of a semidiurnal tidal cycle. Specifically, the control over the Spartel Sill is lost at high water and recovered one hour before low water at Tarifa, and the control over the Camarinal Sill is lost at the end of each half-tidal cycle. The locations of the permanent controls are also time dependent. For instance, the control in the Tarifa Narrows is displaced to the east by an eastward-travelling internal bore, and again returns to its initial position once the bore has passed. Also, the permanent and episodic controls in the strait differ in character of transition from supercritical to subcritical flow: the former are specified by a gentle transition, whereas the latter are specified by a rapid transition, via an internal jump. The reason for this, as stated in Armi and Farmer (1988) and Farmer and Armi (1988), is the change in depth and width. At the Camarinal Sill, the dominant factor is the change in depth, while in the Tarifa Narrows that factor is the change in width.

The model reproduces most of the above-mentioned findings (Figure 3). In particular, (over a tropical month) it predicts the presence of averaged controls to the west of the Spartel Sill, over the Spartel and Camarinal Sills and in the Tarifa Narrows. But, more importantly, it predicts that the average controls to the west of and at the Spartel Sill as well as in the Tarifa Narrows do not extend over the whole width of the strait. Moreover, the latter of these controls splits up into discrete fragments alternating with subcritical flow regions. Strongly speaking, the term ' control ' is not appropriate to that situation because such a fragmentary control cannot provide efficient blocking interfacial disturbances within a subcritical flow region and, hence, cannot completely determine the exchange rate in this region. Therefore, what occurs in the Tarifa Narrows may be referred to, in terms of two-layer hydraulic theory, as an apparent control. Three out of its four fragments are located near the northern and southern shores, and the fourth is situated over the subridge in the central part of the Tarifa Narrows. The control fragment off the southern shore merges with the average controls at the southern portions of the Camarinal and Spartel Sills, forming a single southern control band. The latter reaches westward to the boundary of the model domain. A similar control band arising from junction of the average
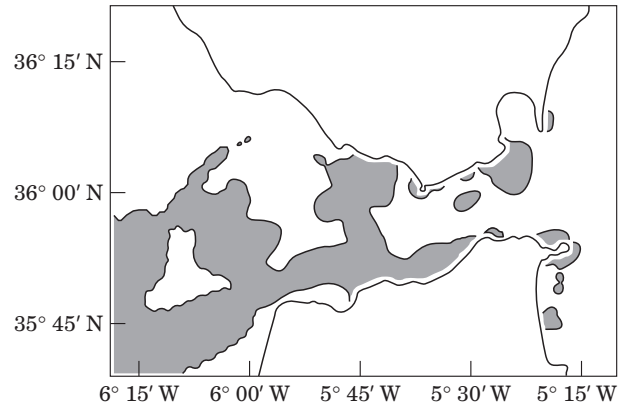

FIGURE 3. Locations of average control in the Strait of Gibraltar. These are defined as the locations for which the Froude number is equal to, or greater than, unity. The Froude number, F, is defined as $F^2 = |\mathbf{u}|^2/gH$ in single-layer regions and $F^2 = \sum_i |\mathbf{u}_i|^2/g'h_i$, $i=1$, 2 in two-layer regions, where $|\mathbf{u}|$ is the modulus of current velocity in a single-layer region; the remaining designations are given in the text. The regions where the above control condition is met are shaded.

controls at the Spartel Sill and in the western part of the model domain is found near the centerline of the strait. As a result, the basin west of the Spartel Sill changes into an isolated subcritical flow basin.

The model also reproduces the tidally-induced periodic loss and subsequent renewal of the controls during the semidiurnal tidal cycle, as well as the concurrent occurrence of the controls at the Camarinal Sill and in the central part of the Tarifa Narrows (Figure 4). These predictions are consistent with the conclusions of Armi and Farmer (1988) and Farmer and Armi (1988). However, the time intervals when the subcritical flow region between the Camarinal Sill and the Tarifa Narrows remains bounded by the corresponding controls appear to be of short duration. For much of a tropical month there is no either or both the controls. That is, our results show that the exchange through the Strait of Gibraltar either switches repeatedly between maximal and submaximal, as has been established by Garret *et al.* (1990), or is not determined by the above controls at all.

Additionally, our results reveal that the time-varying controls, much like the average ones, do not extend over the whole width of the strait. The only exception is the control at the Camarinal Sill section, but during neap tide it also breaks down. It follows that one of the central assumptions underlying internal hydraulic theory, that across-strait variations in flow parameters (two-dimensional effects) can be neglected, does not always hold. This, in turn, implies that, even though the theory involves time dependence and strong tidal forcing, it may be incapable of

describing adequately the time–space variability of hydraulic controls and exchange flows. That is, its applying to not-too-narrow straits with complex geometry, such as the Strait of Gibraltar, seems inappropriate not only due to time dependence, as has been shown by Helfrich (1995), but also due to horizontal two-dimensionality of flow.

Internal bore evolution

The life history of the internal bores in the Strait of Gibraltar inferred from the Gibraltar Experiment data was outlined by Armi and Farmer (1988) and Farmer and Armi (1988). It begins with the formation of an interfacial depression over the western edge of the Camarinal Sill near low water at Tarifa when both layers over the sill crest move west. During the period of approximately one hour before high water, when the lower layer at the sill continues to move west, an internal bore is released from the Camarinal Sill and starts to travel east. Its length scale in the along-strait direction and speed are 3 km and $1 \cdot 7$ m s^{-1}, respectively. Just before next low water, when the flow in both layers is directed to the east, the bore reaches the Tarifa Narrows. Here its speed is increased to $2 \cdot 5$ m s^{-1} by the strong tidal flow in the upper layer. At about the same time, a much weaker westward-travelling internal bore is released from the Camarinal Sill. As these bores move away from the sill, they evolve into dispersive internal wave trains. The eastward-travelling internal bore is generated every semidiurnal tidal cycle during spring tides and on occasion during neap tides. An incidence of westward-travelling internal bores has not been ascertained. The time of bore release from the Camarinal Sill varies over a one-hour range due to the diurnal inequality of the tide. This variability, together with the variability in speed of internal bores due to their difference amplitudes and to variable advection in the upper layer, results in a small variation in arrival time at any locations along the strait. The vertical excursions of the interface accompanying the bore passage are of about 100 m at the Camarinal Sill, 50 m in the Tarifa Narrows and noticeably less just east of the narrowest part of the strait.

Clearly, the above pattern is not comprehensive enough to give an exhaustive account of the internal bores in the Strait of Gibraltar. Therefore, differences between the modelled and observed features are almost inevitable. As an illustration, let us recall that the Brandt *et al*. (1996) model predicts the occurrence of an interfacial depression not only at the Camarinal Sill but also slightly west of the subridges located 7 km west and 4 km east of the sill. Moreover, it predicts

that a small-amplitude internal bore is released from the second of these subridges during weak westward tidal flow. Although such features were not identified by Armi and Farmer (1988) and Farmer and Armi (1988), these have been observed by Watson and Robinson (1990). Since neither any models nor the Gibraltar Experiment data, having inadequate resolution both in the horizontal and within the upper layer, can claim for best describing the internal bore evolution, there is some uncertainty on this point. Below are given the model results that can provide additional information for thoughts on the orderly development of internal bores in the Strait of Gibraltar, including their formation, release and propagation.

Turning to a discussion of the model results, we will first notice that the time of onset of all the above-mentioned events has been given with respect to high water at Tarifa. On evidence derived from deep bottom pressure measurements taken during the Gibraltar Experiment, the Greenwich phase lag of the M_2 surface tide at the station TA south of Tarifa is $41 \cdot 2°$. Then, for example, the release of an eastward-travelling internal bore from the Camarinal Sill falls within the interval between about 0030 and 0130 h of Greenwich Mean Time (GMT). The predicted time of bore release is about 0200 h of GMT. As can be seen from Figure 5, the release of the bore occurs at the tidal phase when the westward flow in the lower layer over the Camarinal Sill has slackened but has not yet reversed its direction. This is also consistent with the observations. For lack of current meter measurements within the upper layer over the Camarinal Sill, the Gibraltar Experiment data do not provide direct evidence on the flow direction in this layer. However, the fact that at the time of bore release the flow in the upper layer moves in a westward direction, as shown in Figure 5, is supported by acoustic and XBT profile measurements (Armi & Farmer, 1988; Farmer & Armi, 1988).

The travel times of the bore from the Camarinal Sill to the Tarifa, Pta. Cires and Gibraltar sections are 2, 4 and 6 h, respectively. It follows that the speed of the bore is $1 \cdot 7$ m s^{-1} between the sill and the Tarifa section, $2 \cdot 5$ m s^{-1} between the Tarifa and Pta. Cires sections, and $1 \cdot 5$ m s^{-1} between the Pta. Cires and Gibraltar sections. As the bore has passed through the Gibraltar section, its speed falls to $1 \cdot 2$ m s^{-1}. The peak-trough amplitude of the bore diminishes progressively eastward from over 70 m at the sill to 60 m at the Tarifa section, 24 m at the Pta. Cires section and 20 m at the Gibraltar section. At the distance of 20 km to the east of Gibraltar, the bore-induced interfacial elevations amount to about 10 m. All these
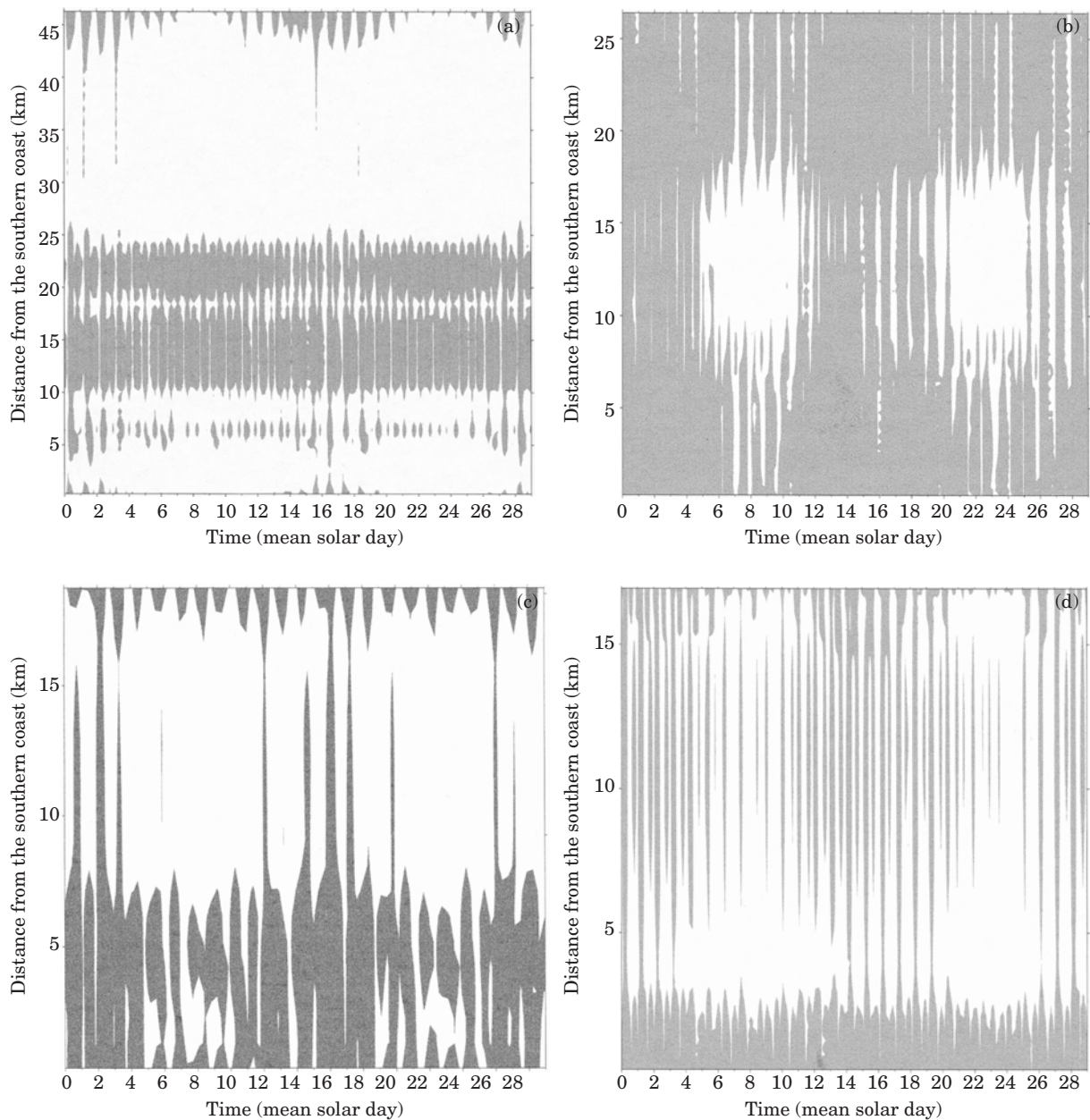
FIGURE 4. (a)–(d).

quantitative estimates as well as the predicted time of onset of one event or another are in good agreement with the observational data of Armi and Farmer (1988) and Farmer and Armi (1988).

Thus, the qualitative pattern of the development of internal bores in the Strait of Gibraltar, as reproduced by the model, is close to the observed one and moreover makes up for its missing details. In particular, the model results depicted in Figure 5 point to a continuous lowering of the interface in the neighborhood of the Camarinal Sill throughout the whole tidal phase when the flow in both layers moves in an eastward direction (between 0500 and 0900 h). This appears to be caused by pure kinematic effects. Indeed, by virtue of the condition of mass conservation, a draining of the Mediterranean water or a filling of the Atlantic water under tidal forcing must be compensated for by opposing changes in the mass of one or the other layer. As a consequence, the large interfacial elevations west of the Camarinal Sill are reduced, and the small interfacial elevations east of the Camarinal Sill are transformed into a depression. In time, this depression becomes deeper, so that by 0800 h the flows east and west of the sill are matched
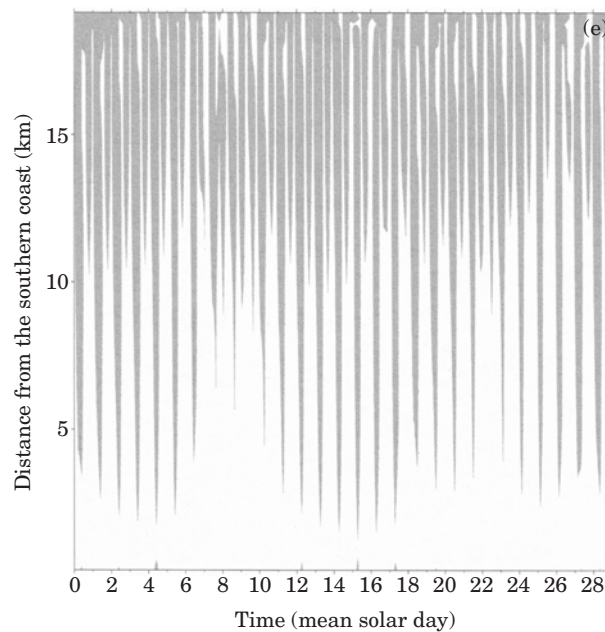
FIGURE 4. (e).

FIGURE 4. Variations of the Froude number over a tropical month and in the across-strait direction. The panels correspond to the cross sections through the Spartel (a) and Camarinal (b) Sills as well as the western (c), central (d) and eastern (e) parts of the Tarifa Narrows. In Figure 2 these cross sections are shown in order of their mentioning from west to east. The regions where the Froude number is equal to, or greater than, unity are shaded.

with a hydraulic jump. Towards the end of the stage of eastward flow (0900 h), the jump turns into a hydraulic jump-drop pair. At the same time a small-amplitude monotonic internal bore released from the Camarinal Sill begins to travel in a westward direction. Its peak–trough amplitude increases during three hours and then it decreases as the westward flow in both layers progresses.

Between 1100 and 0100 h, when the transport of the Mediterranean water to the west increases, the interface over the Camarinal Sill and its vicinities rises, resulting in the formation of the bulges of lower-layer fluid on each side of the sill. This is accompanied by a partial degradation of the depression and its slight displacement upstream. At 0200 h, just after flooding the control at the Camarinal Sill, the depression located west of the eastern bulge begins to develop into a large-amplitude internal bore which is released from the sill and travels upstream (to the east). Whereas, the western bulge is arrested and generates a small-amplitude bore-like interfacial disturbance in the lee of the Spartel Sill. This disturbance, like a lee wave travelling upstream at phase velocity equal to background velocity, remains stationary and keeps on growing for approximately 2 h until the flow in both layers reverses its direction. Thereafter, when the eastward flow is initially

strengthened and then slackens, the disturbance starts travelling upstream (to the west). During the whole considered period (from 0300 h and later), the eastward-travelling internal bore moves without essential changes in shape and gradually decays. The above sequence of events is repeated every semi-diurnal tidal cycle with some variations at spring and neap tide periods.

Also shown in Figure 5 are along-strait profiles of the interface depth relative to its mean value for the case of no rotation. It is known that the earth's rotation influences interfacial disturbances and can prevent them from disintegrating into oscillatory wave trains if, for sufficient strong non-linearity, length scales of these disturbances are of the order of, or greater than, the internal Rossby radius of deformation (Gerkema & Zimmerman, 1995, see also Tomasson & Melville, 1992, and Grimshaw et al., 1998). Our model does not include non-hydrostatic dispersion counteracting the amplitude dispersion. Consequently, the neglect of the effect of rotation, one more stabilizing factor, must seemingly enhance the non-linear tendency for growing and steepening of interfacial disturbances. This can indeed be noted in an increased steepness of the westward-travelling internal bore, a decreased number of non-linear internal waves behind the eastward-travelling internal
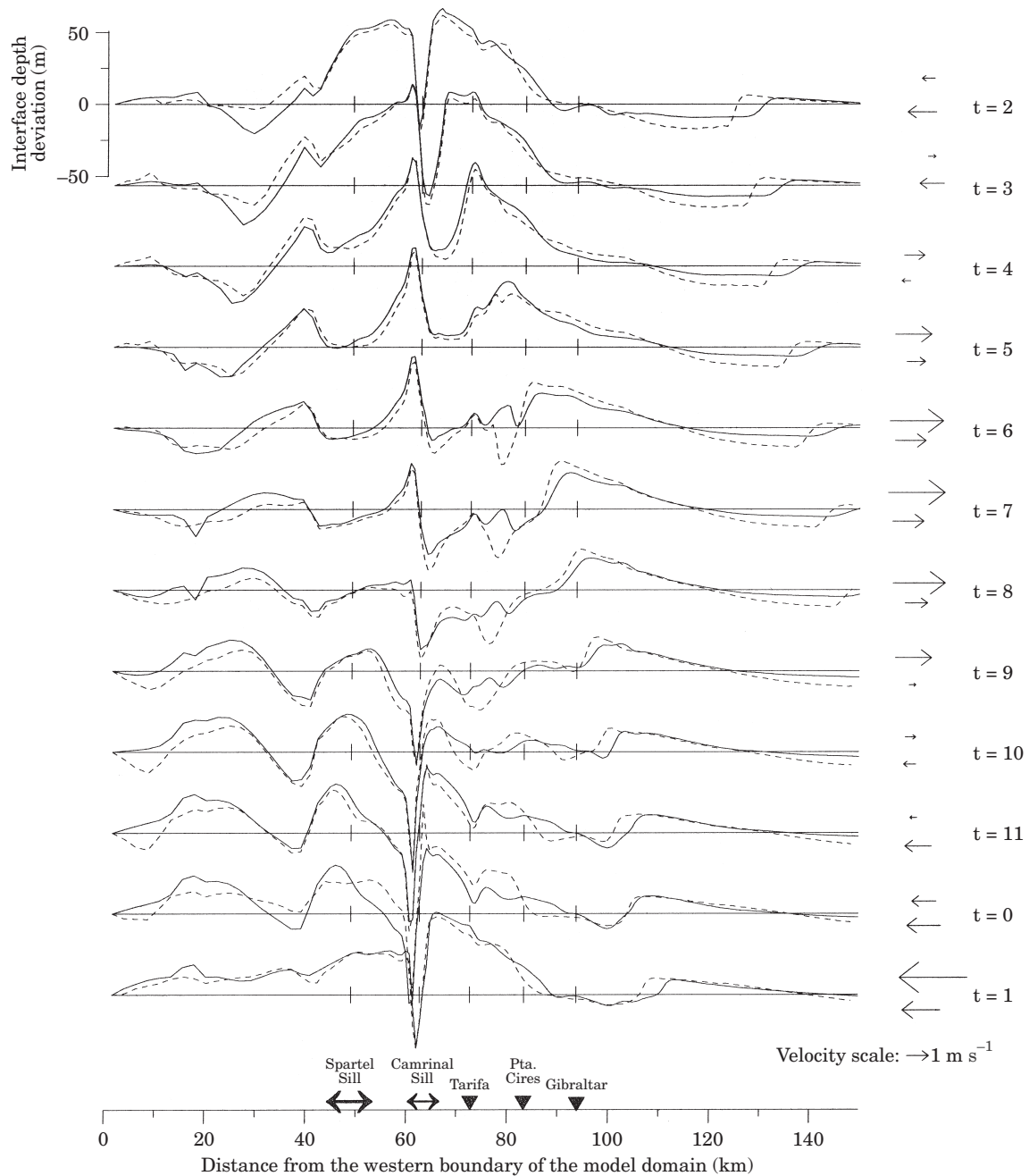
FIGURE 5. Deviation of the interface depth from its mean value as a function of the distance from the western boundary of the model domain at various times of a semidiurnal tidal cycle for spring tide. Presented here are the calculation results appropriate to the along-strait section shown in Figure 2. Solid and dashed lines correspond to the case with and without rotation, respectively. The arrows indicate the direction and magnitude of the current velocity in the upper and lower layer over the Camarinal Sill. Time, *t*, is given in mean solar hours, with *t*=0 being made coincident with 0000 h GMT.

bore, and a reversed polarity of some of these waves. Nevertheless, the two sets of curves are very similar. This is because the model involves other determining factors, such as geometric spreading, variable bottom topography and friction, which may have roles in suppression of the disintegration process on the length

scales of internal bores that are no less significant than the role of non-hydrostatic dispersion.

That geometric spreading is of great concern in the development of internal bores in the Strait of Gibraltar is readily visualized from Figure 6. Presented here are the predicted fields of the surface
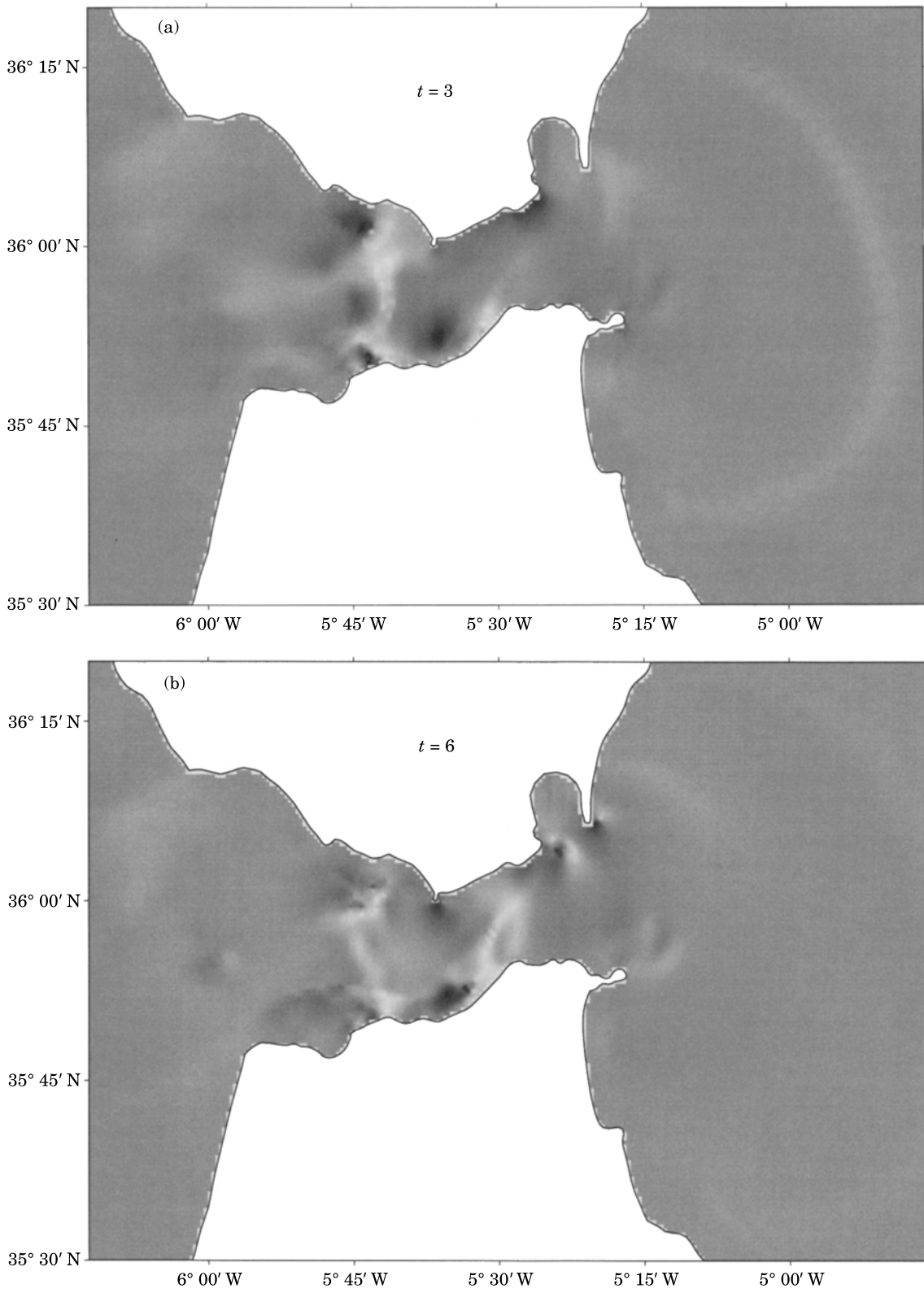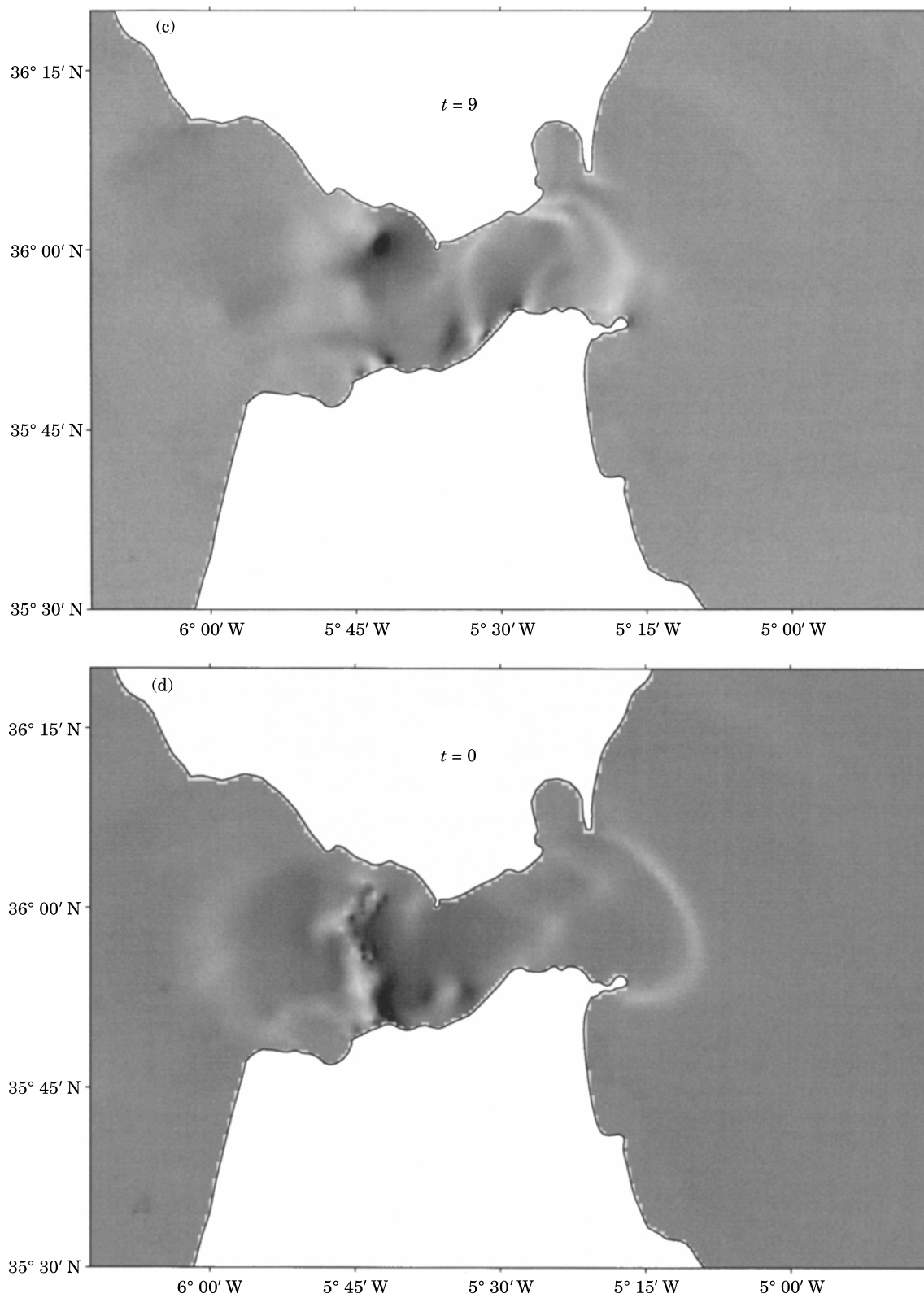
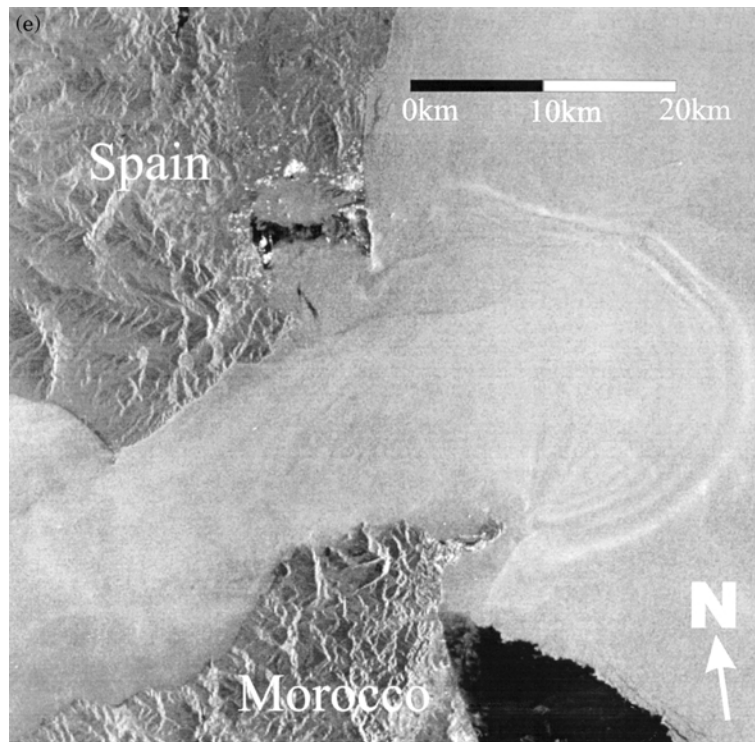FIGURE 6. (a) and (b). Caption on page 649.

FIGURE 6. (c) and (d).

FIGURE 6. (e).

FIGURE 6. Spatial structure of internal bores in the Strait of Gibraltar at various times of a semidiurnal tidal cycle for spring tide. The panels (a), (b), (c) and (d) show the model predictions of the surface strain; the panel (e) is the ERS-1 SAR image (6 February, 1992, 1104 UTC, orbit 2929, frames 2871/2889) of surface signatures of an eastward-travelling internal bore. The bore front is shown by lighter shading. The time, t is given in mean solar hours.

strain at various times of a semidiurnal tidal cycle for spring tide and the surface signatures of an eastward-travelling internal bore inferred from the ERS-1 SAR image of 6 February 1992, 1104 UTC. As can be seen, the model and the ERS-1 SAR imagery provide nearly identical patterns of bore front. However, our model does not reproduce another important feature detected in the ERS-1 SAR image, the splitting of the internal bore into internal solitary waves. The reason is that, while the model includes dispersive factors, it nevertheless ignores non-hydrostatic dispersion and, hence, cannot capture the above phenomenon. Following the Gerkema and Zimmerman (1995) conclusion, this is because geometric spreading, variable bottom topography and rotation determine the overall structure, while non-hydrostatic dispersion operating on the length scales of internal solitary waves determines the fine structure of internal bores.

Summary and conclusions

This paper has studied the time–space variability of hydraulic controls and the development of internal bores in the Strait of Gibraltar using, as our primary tool, a 2-D high-resolution, non-linear, two-layer, free-surface, boundary-fitted co-ordinate, hydrostatic model. The model results show that there are four averaged (over a tropical month) controls located to the west of the Spartel Sill, at the Spartel and Camarinal Sills, and in the Tarifa Narrows. The average control in the Tarifa Narrows comprises nothing more than an apparent control in the sense that it consists of discrete fragments alternating with subcritical flow regions.

The model has also reproduced the tidally-induced periodic loss and renewal of the controls during a semi-diurnal tidal cycle and, accordingly, a tropical month. In both the cases of average and time-varying control, the only control which extends over the whole width of the strait is that located at the Camarinal Sill, but it breaks down during neap tide, too. Moreover, the controls at the Camarinal Sill and in the central part of the Tarifa Narrows occur concurrently for short periods, while for much of the tropical month there is either just one or neither of the controls. It follows that the exchange in the Strait of Gibraltar either switches repeatedly between maximal and sub-maximal, as has been argued by Garrett et al. (1990),

or is not determined by these controls at all. Another conclusion to emerge from the model results is that one of the central assumptions underlying internal hydraulic theory, the assumption of the possibility of horizontal two-dimensionality of flow being neglected, can fail in not-too-narrow straits with complex geometry, like the Strait of Gibraltar.

The model has been demonstrated to provide not only a general pattern of the development of internal bores in the strait in good agreement with that deduced from observational data, but also its missing details. Of particular interest are the appearance of the bulges of Mediterranean water to the east and west of the Camarinal Sill; and the evolution of an eastward-travelling, large-amplitude bore and a small-amplitude, bore-like interfacial disturbance in the lee of the Spartel Sill.

The model results indicate that the neglect of earth's rotation does lead to a small enhancing of the non-linear tendency for growing and steepening of interfacial disturbances over only a period of several hours, rather than throughout the whole semi-diurnal tidal cycle. This finding is attributed to the influence of other determining factors, such as geometric spreading, variable bottom topography and friction, the significance of which on the length scales of internal bores may be no less than the significance of non-hydrostatic dispersion. The fact that geometric spreading markedly affects the development of internal bores in the Strait of Gibraltar is confirmed by the similarity between the predicted spatial structure of bore front and the internal bore signatures inferred from the ERS-1 imagery.

Finally, the authors stress that, for ignoring non-hydrostatic dispersion which may dominate other determining factors on the length scales of internal solitary waves and, besides, including dissipative mechanisms of interfacial and bottom friction, their model precludes the presence of internal solitary waves. This limitation may be regarded as the price to be paid in order to obtain a more realistic description of the spatial structure of flow, which has made it possible to reveal some new aspects of control behaviour and internal bore evolution in the Strait of Gibraltar.

Acknowledgements

References

Alpers, W. 1985 Theory of radar imaging of internal waves. *Nature* **314,** 245–247.

Alpers, W. & La Violette, P. E. 1993 Tide-generated nonlinear internal wave-packets in the Strait of Gibraltar observed by the synthetic aperture radar aboard the ERS-1 satellite. *Proceedings of the First ERS-1 Symposium*, Eur. Space Agency Special Publication, ESA-SP **359,** 753–758.

Armi, L. & Farmer, D. M. 1986 Maximal two-layer exchange through a contraction with barotropic net flow. *Journal of Fluid Mechanics* **164,** 27–51.

Armi, L. & Farmer, D. M. 1988 The flow of Mediterranean Water through the Strait of Gibraltar. *Progress in Oceanography* **21,** 1–105.

Brandt, P., Romeiser, R. & Rubino, A. 1999 On the determination of characteristics of the interior ocean dynamics from radar signatures of oceanic internal solitary waves. *Journal of Geophysical Research* **104,** 30039–30046.

Brandt, P., Alpers, W. & Backhaus, J. O. 1996 Study of the generation and propagation of internal waves in the Strait of Gibraltar using a numerical model and synthetic aperture radar images of the European ERS-1 satellite. *Journal of Geophysical Research* **101,** 14237–14252.

Bryden, H. L. & Kinder, T. H. 1991 Steady two-layer exchange through the Strait of Gibraltar. *Deep-Sea Research* **38** (Supplement 1),5445–5463.

Bryden, H. L., Candela, J. & Kinder, T. H. 1994 Exchange through the Strait of Gibraltar. *Progress in Oceanography* **33,** 201–248.

Cavanie, A. G. 1972 Observations de fronts internes dans le Détroit de Gibraltar pendant la Campagne Océanographique OTAN 1970 et interpretation des résultats par un modéle mathématique. *Mémoires de la Societe des Sciences de Liége* **6,** 27–41.

Deacon, M. 1985 An early theory of ocean circulation: J. S. Von Waitz and his explanation of the currents in the Strait of Gibraltar. *Progress in Oceanography* **14,** 89–101.

Farmer, D. M. & Armi, L. 1986 Maximal two-layer exchange over a sill and through the combination of a sill and contraction with barotropic flow. *Journal of Fluid Mechanics* **164,** 53–76.

Farmer, D. M. & Armi, L. 1988 The flow of Atlantic water through the Strait of Gibraltar. *Progress in Oceanography* **21,** 1–105.

Frassetto, R. 1964 Short period vertical displacements of the upper layers in the Strait of Gibraltar. *Technical Report* 30, SACLANT ASW Research Centre, La Spezia, Italy, 49 pp.

Garrett, C., Bormans, M. & Thompson, K. 1990 Is the exchange through the Strait of Gibraltar maximal or submaximal? In *The Physical Oceanography of Sea Straits* (Pratt, L. J., ed.). Kluwer Academic Publishers, Dordrecht, The Netherlands.

Gerkema, T. & Zimmerman, J. T. F. 1995 Generation of nonlinear internal tides and solitary waves. *Journal of Physical Oceanography* **25,** 1081–1094.

Grimshaw, R., Ostrovsky, L. A., Shira, V. I., Stepanyants, V. I. & Yu, A. 1998 Long nonlinear surface and internal gravity waves in a rotating ocean. *Surveys in Geophysics* **19,** 289–338.

Helfrich, K. R. 1995 Time-dependent two-layer hydraulic exchange flows. *Journal of Physical Oceanography* **25,** 359–373.

Helfrich, K. R. & Melville, W. K. 1990 Review of dispersive and resonant effects in internal wave propagation. In *The Physical Oceanography of Sea Straits* (Pratt, L. J., ed.). Kluwer Academic Publishers, Dordrecht, The Netherlands.

Instituto Geográfico Nacional y SECEG 1988 *Estrecho de Gibraltar*. Instituto Geográfico Nacional, General Ibañez de Íbero, 3-28071, Madrid, Spain.

Kinder, T. H. 1984 Net mass transport by internal waves near the Strait of Gibraltar. *Geophysical Research Letters* **11,** 987–990.

Kropfli, R. A., Ostrovskii, L. A., Stanton, T. R., Skirta, E. A., Keane, A. N. & Irisov, V. 1999 Relationships between strong internal waves in the coastal zone and their radar and radiometric signatures. *Journal of Geophysical Research* **104,** 3133–3148.

Lacombe, H., Tchernia, P., Richez, C. & Gamberoni, L. 1964 Deuxiéme contribution á l'etude du régime du Détroit de Gibraltar. *Cahiers Océanographiques* **16,** 283–327.

La Violette, P. E. & Arnone, R. A. 1988 A tide-generated internal waveform in the western approaches to the Strait of Gibraltar. *Journal of Geophysical Research* **93,** 15653–15667.

La Violette, P. E., Kinder, T. H. & Green, D. W. 1986 Measurements of internal waves in the Strait of Gibraltar using a shore-based radar. *Technical Report* **118,** Naval Ocean Research and Development Activity, National Space Technology Laboratory, Bay of St. Louis, MS, U.S.A., 13 pp.

Le Provost, C., Lyard, F., Molines, J. M., Genko, M. L. & Rabillloud, F. 1998 A hydrodynamic ocean tide model improved by assimilating a satellite altimeter-derived data set. *Journal of Geophysical Research* **103,** 5513–5529.

Longo, A., Manzo, M. & Pierini, S. 1992 A model for the generation of nonlinear internal tides in the Strait of Gibraltar. *Oceanologica Acta* **15,** 233–243.

Richez, C. 1994 Airborne synthetic aperture radar tracking of internal waves in the Strait of Gibraltar. *Progress in Oceanography* **33,** 93–159.

Romeiser, R. & Alpers, W. 1997 An improved composite surface model for the radar backscattering cross-section of the ocean surface. 2. Model response to surface roughness variations and the radar imaging of underwater bottom topography. *Journal of Geophysical Research* **102,** 25251–25267.

Sein, D. V., Backhaus, J. O., Brandt, P., Izquierdo, A., Kagan, B. A., Rubino, A. & Tejedor, L. 1999 Flow exchange and tidally induced dynamics in the Strait of Gibraltar derived from a two-layer, boundary-fitted coordinate model. In *Oceanic Fronts and Related Phenomena* (Konstantin Fedorov Memorial Symposium). IOC Workshop Report Series, **159,** UNESCO'99.

Thomson, D. R. 1988 Calculation of radar backscatter modulations from internal waves. *Journal of Geophysical Research* **93,** 12371–12380.

Tomasson, G. G. & Melville, W. K. 1992 Geostrophic adjustment in a channel: nonlinear and dispersive effects. *Journal of Fluid Mechanics* **241,** 23–57.

Watson, G. & Robinson, I. S. 1990 A study of internal wave propagation in the Strait of Gibraltar using shore-based radar images. *Journal of Physical Oceanography* **20,** 374–395.

Wesson, J. C. & Gregg, M. C. 1994 Mixing at Camarinal Sill in the Strait of Gibraltar. *Journal of Geophysical Research* **99,** 9847–9878.

Ziegenbein, J. 1969 Short internal waves in the Strait of Gibraltar. *Deep-Sea Research* **16,** 479–487.

Ziegenbein, J. 1970 Spatial observations of short internal waves in the Strait of Gibraltar . *Deep-Sea Research* **17,** 867–876.