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The role of turbulence in the sedimentation loss of pelagic aggregates from the mixed layer

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ABSTRACT

A simple approach is presented to model the effect that turbulence has on the sedimentation loss of pelagic particles from the mixed layer. The approach consists in determining how turbulence affects their vertical distribution by splitting the solution of the advection-diffusion equation into two functions, one for the concentration of particles in the mixed layer and the other one reflecting the shape of the profile of particle concentration. The results of the paper indicate that the sedimentation flux is seriously underestimated if a uniform distribution of particles is assumed in the mixed layer when turbulence levels are low. A correction to this underestimation is possible in many situations without resolving the vertical scale in the mixed layer. The cases in which the correction cannot be applied are delimited in the paper in terms of dimensionless numbers. The results also demonstrate the importance of resolving the vertical scale in models of particle dynamics and add further support to the hypothesis of turbulence as the mechanism responsible for daily cycles of aggregates in the sea.

1. Introduction

The presence of turbulence in the ocean surface mixed layer has important consequences for numerous biological processes of the pelagic ecosystem such as photosynthesis (Belyaev, 1992) or grazing (Saiz *et al.,* 1992). A process that one might expect to be affected by the presence of turbulence in the mixed layer is the rate at which particles are exported via sedimentation. In fact, the assumption that turbulence decreases the rate at which phytoplankton cells are exported from the ocean surface mixed layer has been one of the paradigms of pelagic ecology but with little direct empirical or theoretical support.

The analyses of the effect that turbulence has on settling particles have been mainly limited to studying how different is their diffusion coefficient from that of the surrounding fluid (Reeks, 1977; Nir and Pismen, 1979; Mei *et al.,* 1991; Young and Hanratty, 1991). First theoretical developments (Yudine, 1959) indicated the presence of what is known as the Yudine or crossing trajectories effect (Csanady, 1963; Wells and Stock, 1983), i.e., a decrease of the diffusion coefficient of the settling particle when compared with that of the fluid. Recent studies, based on a very

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detailed analysis and modeling of the forces acting on a particle in a turbulent flow, have focused attention on how the presence of turbulence affects the settling velocity of particles (Wang and Maxey, 1993; Fung, 1993). These studies show that when the Stokes' number of a particle is very small its average settling velocity in a turbulent flow is not different from that of the same particle in a still fluid. Stokes' numbers for pelagic particles are, as will be shown in this paper, very small. Consequently, if turbulence has any effect on the rate at which pelagic particles are exported from the mixed layer, it will not be because it alters their settling velocity.

Lande and Wood (1987) proposed a different approach to modeling how turbulence affects particle sedimentation on the basis of stochastic theory. Their model is able to predict an increase in the residence time of phytoplankton cells at the ocean surface mixed layer if they decrease their settling velocity at the top few meters of the thermocline. That decrease is expected for phytoplankton cells because it has been reported that phytoplankton settling velocity decreases when light decreases (Bienfang *et al.,* 1983). However, their model is unsuitable for detritus or phytoplankton cells that are either large or are in aggregates and therefore have fast settling velocities. In these cases one does not expect that the physiological changes producing a decrease in the settling velocity of phytoplankton takes place in the top few meters of the thermocline as the model of Lande and Wood requires. The change in the physical characteristics of the sea water, encountered by the particle when sinking through the thermocline, can produce instantaneous changes in the settling velocities of particles. If the particle is sinking according to Stokes' law, the changes in density of sea water will have a negligible effect as the relative changes of density are of the order of 10^{-3}. Changes in the dynamic viscosity of sea water can only reduce the settling velocity of particles to approximately 80% its velocity in the mixed layer (for a decrease of 10 degrees along the thermocline and for particles sinking according to Stokes' law). Without a substantial decrease in the settling velocity of particles, the model of Lande and Wood (1987) predicts a small effect of turbulence on the residence time of particles.

The approach followed in this paper is different from the one presented by Lande and Wood in that, although both make use of diffusion theory, the effect of turbulence is studied by analyzing the consequences that its presence has on the spatial distribution of particles in the mixed layer, rather than modeling individual particles through stochastic theory. The advantages of this approach are its simplicity and the fact that the advection-diffusion equation has already been successfully used to predict the vertical distribution of particles sinking in a turbulent flow (Pasquill, 1962; Graf, 1971; Csanady, 1973; Okubo, 1980; Pruppacher and Klett, 1978; Lau, 1989). The variety of situations in which the advection-diffusion model has been able to predict the vertical distribution of particles makes it sensible to expect that it can also be successfully used to predict the spatial distribution of aggregates in the mixed layer.

The disadvantage of the advection-diffusion approach is that it requires resolving the vertical scale for the distribution of particles in the mixed layer. In this paper an approach akin to self-similarity theory of decaying turbulence (Lesieur, 1993) or aerosols size-spectrum (Friedlander and Wang, 1966) that permits avoiding this handicap in many situations is presented. Our approach, however, only gives a partial solution to the problem since aggregation may alter (in some situations) the vertical distribution of large aggregates that is expected from only the advection and diffusion terms. The situations in which this approach is not reliable can, nevertheless, be identified in terms of dimensionless numbers. The results of this paper also demonstrate the importance of resolving the vertical scale in models of particle dynamics and add further support to the hypothesis of turbulence as the mechanism responsible for daily cycles of aggregates in the sea (Ruiz, 1996).

2. The advection-diffusion approach

a. Boundary value problem

Many ecosystems (Evans and Parslow, 1985; Fasham *et al.,* 1990; Fasham, 1996) or particle models (Jackson, 1990; Jackson and Lochmann, 1992, Riebesell and Wolf-Gladrow, 1992; Hill, 1992; Ruiz, 1996) work with the idealization of a well-mixed layer with a uniform concentration of particles. The export of particles from the mixed layer is then modeled by defining its loss rate as:

$$\left[\frac{d\overline{C}}{dt}\right]_{\mathrm{Sed}} = -\frac{w}{h}\overline{C} \tag{1}$$

where w is the settling velocity of the particle, h is the mixed layer depth, t is time and $\overline{C}$ is the average number or mass concentration of particles in the mixed layer.

The consideration of a uniform profile of particles in the mixed layer is an idealization that in some cases (as will be shown later) is far from the real case. It is, however, a necessary assumption in certain cases, otherwise it would be necessary to resolve the vertical scale of particle concentration in the mixed layer and the computational effort needed to integrate models would, consequently, increase strongly. An approach closer to reality but without resolving spatial nonuniformity is to consider that particles are in the mixed layer in a state in which (despite variations in the total number of particles in the layer) the shape of the profile of particle concentration is constant in time. When this profile is reached, the concentration of particles at the bottom of the mixed layer might differ from the average concentration of particles in the layer. As the particles that are lost from the mixed layer are those sedimenting from its bottom, the more the concentration of particles in the bottom differs from the average concentration the more inaccurate will be the loss term that considers a uniform distribution of particles.

To study this problem we must analyze the advection-diffusion equation that governs the vertical distribution of settling particles in a turbulent flow. This equation is:

$$\frac{\partial C}{\partial t} = -w\frac{\partial C}{\partial z} + K\frac{\partial^2 C}{\partial z^2} \tag{2}$$

where K is vertical eddy diffusivity and z is depth. The boundary conditions for this equation are that there is no flux at the surface (a sensible condition for pelagic aggregates) and that the flux at the bottom of the mixed layer is due to the sedimentation of particles through the thermocline. Then, the boundary conditions are:

$$\begin{aligned} -wC + K\frac{\partial C}{\partial z} &= 0 \qquad \text{at } z = 0 \\ -wC + K\frac{\partial C}{\partial z} &= -wC \qquad \text{at } z = h \end{aligned} \tag{3}$$

where h is the depth of the mixed layer. To cast Eqs. (2) and (3) in dimensionless form, we define a dimensionless time (τ), depth (η), and diffusivity (D) as:

$$\tau = t\frac{w}{h} \qquad \eta = \frac{z}{h} \qquad D = \frac{K}{wh}. \tag{4}$$

The dimensionless version of Eq. (2) and conditions (3) is displayed in Eq. (5). The analytical solution of Eq. (5) is given in Appendix 1.

$$\begin{aligned} &\frac{\partial C}{\partial \tau} = -\frac{\partial C}{\partial \eta} + D\frac{\partial^2 C}{\partial \eta^2} \\ &-C + D\frac{\partial C}{\partial \eta} = 0 \qquad \text{at } \eta = 0 \\ &\frac{\partial C}{\partial \eta} = 0 \qquad \text{at } \eta = 1 \end{aligned} \tag{5}$$

b. The necessity to correct the sedimentation term

The solution to Eq. (2) with the boundary conditions (3) does not possess a steady state solution as there is a constant sinking of particles from the mixed layer and, therefore, a constant decrease of particles. However, the shape of the distribution of particles in the mixed layer does indeed have the possibility of a steady state. This problem can be better studied if we follow an approach akin to that of self-similarity theory of turbulence (Lesieur, 1993) or of aerosols size-spectrum (Friedlander and Wang, 1966). Thus, the solution of Eq. (5) is split into two functions: a function for the concentration and another for the shape of the vertical profile of particles. For the first function, the concentration of aggregates per unit area in the mixed layer,

$A(\tau)$, will be used. For the second function the solution of Eq. (5) divided by $A(\tau)$ will be used, this function is named as $\psi(\eta, \tau)$. Then:

$$C(\eta, \tau) = A(\tau)\left(\frac{C(\eta, \tau)}{A(\tau)}\right) = A(\tau)\psi(\eta, \tau)$$
$$\psi(\eta, \tau) \equiv \frac{C(\eta, \tau)}{A(\tau)}. \tag{6}$$

The integral of $\psi(\eta, \tau)$ over the mixed layer is independent of time and its numerical value is always 1. This function represents the way the particles distribute in the mixed layer independently of their actual concentration. An expression for $A(\tau)$ can be obtained by integration of the solution to the boundary value problem (5). The analytical expression of this integral is shown in Appendix 2 for the case of an initial uniform particle distribution in the mixed layer.

Although the solution to the boundary value problem (5) does not reach a steady state, $\psi(\eta, \tau)$ reaches an asymptotic state that is representative of how particles distribute within the mixed layer under certain turbulence conditions. This feature is displayed in Figure 1. When this asymptotic state is reached, $\psi(\eta, \tau)$ does not depend on τ but is only a function of η. As we can see in this figure, the asymptotic form of $\psi(\eta, \tau)$ varies depending on the dimensionless diffusivity (the inverse of the Peclet number), which is the key parameter controlling its shape. For high values of D ($D > 1$) the distribution of particles in the mixed layer is uniform (not displayed). Therefore, in these cases there is little error when assuming a loss rate that is based on the average concentration of particles in the mixed layer. The value of $D = 1$ marks the point in which the error in the estimation of the export of particles from the mixed layer, resulting from considering a uniform distribution of particles, becomes significant. For smaller dimensionless diffusivities, the distribution of particles becomes highly heterogeneous and most of the particles are at the bottom of the mixed layer for diffusivities of the order of 0.01 or lower. In order to obtain the curves displayed in Figure 1, the analytical solutions derived in Appendices 1 and 2 are used; except for the curves corresponding to dimensionless times smaller than 2 in the figure corresponding to $D = 0.005$. It was not possible to use the analytical solution in these cases because some of the terms in the summatory involved very large (positive and negative) numbers and round off error made the series not converge to a right answer. In these cases we used numerical integration to find the curves. As the integral over 0 and 1 (therefore, over the dimensionless depth of the mixed layer) of $\psi(\eta, \tau)$ is always 1, the average value of $\psi(\eta, \tau)$ in the mixed layer, $\overline{\psi}$, is 1 (the average value of a function in an interval is its integral over that interval divided by the width of the interval). The ratio (F) of the concentration at the bottom to the average concentration of particles in the mixed layer will be numerically equal to $\psi(1, \tau)$ but without dimensions since $F = \psi(1, \tau)/\overline{\psi}$. This ratio is the times we are

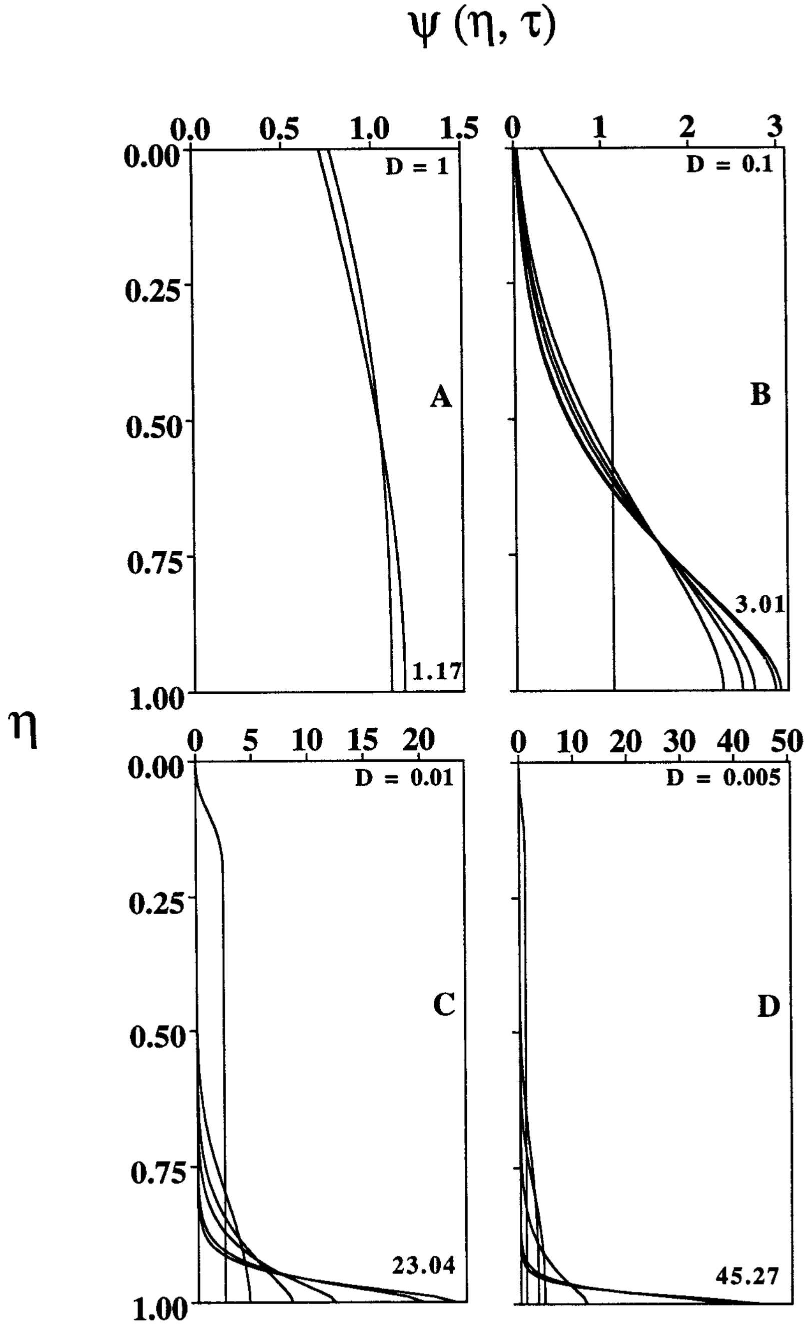

Figure 1. Evolution of $\psi(\eta, \tau)$ in the mixed layer from a uniform distribution to an asymptotic state that depends on D. In all the figures the different dimensionless times represented are: 0.1, 0.8, 1, 1.2, 2 and 3. In Figure A the different curves corresponding to the dimensionless times 0.8, 1, 1.2, 2 and 3, although plotted, cannot be distinguished. The value of $\psi(1, 3)$ is displayed at the bottom of each figure.

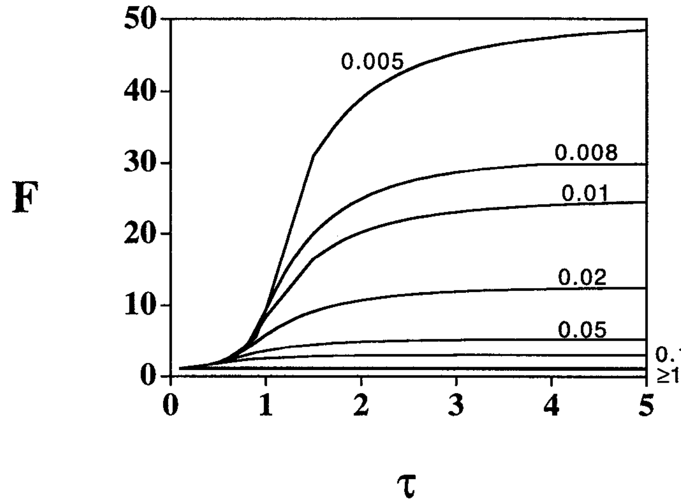

Figure 2. Process of asymptotization of the correction factor (F) for different values of dimensionless diffusivities (D). The numbers marking the curves are the values of D.

underestimating the export rate of particles from the mixed layer by assuming a uniform distribution of particles in that layer. Therefore, it can be used to correct the sedimentation rate to a more realistic one that accounts for the nonuniform distribution of certain particles in the mixed layer. The corrected sedimentation term would then be:

$$\left[\frac{d\overline{C}}{dt}\right]_{\mathrm{Sed}} = -\frac{w}{h}F\overline{C}. \tag{7}$$

The evolution in time of the correction factor is displayed in Figure 2 for the case of an initial uniform distribution. In this figure we can see how the factor tends asymptotically toward a value that depends on the dimensionless diffusivity. A feature that can also be seen in this figure is that F stabilizes after a dimensionless time of approximately two, independent of D. Therefore, this time can be taken (after transforming it into a dimensional one) as a time scale of the time taken for the shape of the profile of particle concentration to reach an asymptotic state in the mixed layer. Another characteristic observable in Figure 2 is that the dependence of the correction factor F on D is not linear and that, when the value of D is low, small changes in D produce big changes in F. A feature that is also clearly apparent in Figure 3, in which the asymptotic value of the correction factor is plotted versus the value of dimensionless diffusivity. The range of D for particles of different sizes is displayed in Figure 4 in a size spectrum fashion. This range was calculated by considering two pairs of h and K (leading to a high and a low value of D) and combining it with the settling velocities of the aggregates as obtained from the model

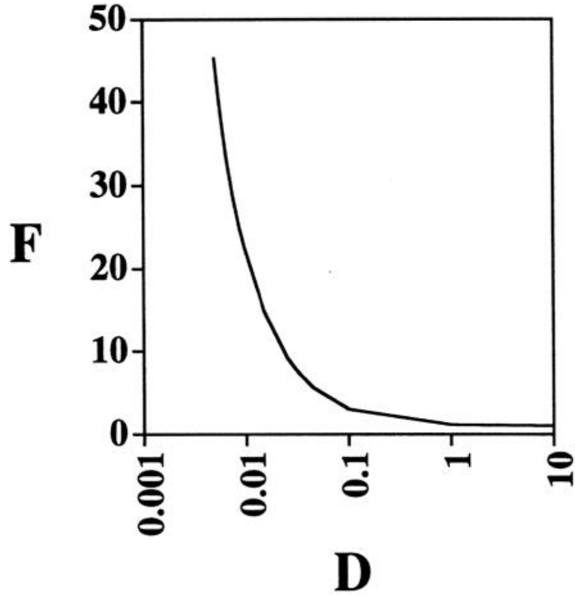

Figure 3. Dependence of the correction factor (F) on dimensionless diffusivity (D).

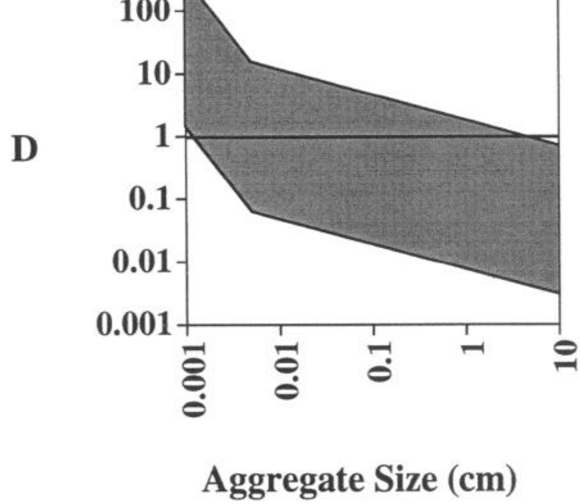

Figure 4. Range of variation of the dimensionless diffusivity (D) in the mixed layer as a function of aggregate size.

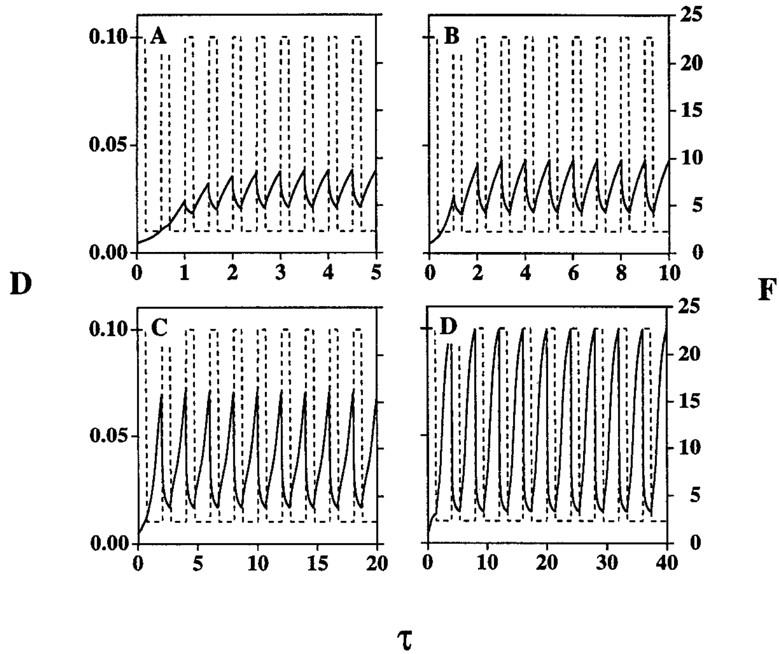

Figure 5. Effect of daily oscillations of dimensionless diffusivity (D) on the correction factor (F). The broken and solid lines respectively represent the evolution of D and F. The scales in the left axis correspond to values of D and the scales on the right axis correspond to the values of F. The values of the oscillating D were purposely held constant in the different figures to make the comparisons easier among different dimensionless frequencies of oscillation. The different dimensionless frequencies of oscillation are: (A) 2; (B) 1; (C) 0.5; (D) 0.25. The different situations producing the frequencies and diffusivities displayed in this figure are indicated in Table 1.

of Ruiz (1996). This model predicts that the settling velocity of an aggregate is related to its radius according to the following expression $w = (\delta\rho/18\mu)(1 - P)gs^2$; where P is the aggregate porosity, s is the diameter, g is gravitational acceleration, and $\delta\rho$ is the density difference between the matter forming the aggregate and the sea water. The combinations of h and K were (h = 40 m, K = 10000 m^2 d^{-1}) and (h = 100 m, K = 100 m^2 d^{-1}). The two different slopes observed in Figure 4 appear as a result of the change with size of the porosity of the aggregates (Ruiz, 1996).

c. Daily cycles of turbulence

The effect of daily cycles of turbulence in the mixed layer (Brainerd and Gregg, 1993) on the rate of sedimentation loss of aggregates is investigated in Figure 5. In this figure, the evolution of the correction factor F is represented when dimensionless diffusivities alternate between 0.1 and 0.01 in a daily cycle. The key parameter controlling the amplitude of the oscillations generated by the oscillating turbulence

Table 1. An example of different combinations of w, h and oscillating eddy diffusivities that produce the dimensionless diffusivities and frequencies of oscillation displayed in Figure 5.

Case	w (m d^{-1})	h (m)	K_1 (m^2 d^{-1})	K_2 (m^2 d^{-1})
A	50	100	50	500
B	100	100	100	1000
C	100	50	50	500
D	200	50	100	1000

level is the dimensionless frequency of oscillation. As the frequency of oscillation of the vertical eddy diffusivity in the mixed layer is 1 d^{-1} (Brainerd and Gregg, 1993), the dimensionless frequency of oscillation is (1 d^{-1}) (h/w). We saw above that the dimensionless time taken for the shape of the distribution of particles in the mixed layer to become asymptotic is approximately 2. Therefore, it is logical to expect that when the dimensionless frequency of oscillation is lower than 1/2 the factor will respond to oscillating turbulence levels in the mixed layer by oscillating between its asymptotic values corresponding to the values of D for day and night. For dimensionless frequencies higher than 1/2 the factor will oscillate between intermediate values and will never reach the asymptotic values that correspond to the oscillating diffusivities. The higher the frequency of oscillation the less able the factor is to reach the asymptotic value corresponding to each diffusivity (day and night). This feature is clearly apparent in Figure 5. In this figure, the different cases represent different dimensionless frequencies of oscillation while the value of the oscillating D is kept constant in all cases (to ease comparisons among different frequencies). The different situations producing the frequencies and oscillating diffusivities in Figure 5 are indicated in Table 1.

d. *The crossing trajectories effect*

The decrease in the turbulence levels that occurs daily in the mixed layer during daytime makes the magnitude of the turbulent velocities also oscillate with a daily cycle. If the magnitude of these velocities is low enough as to be comparable to the settling velocities of the aggregates, the diffusivity of the particles will be lower than the diffusivity of the surrounding fluid because of the crossing trajectories effect (Csanady, 1963; Wells and Stock, 1983; Wang and Stock, 1993). This is a phenomenon that also depends on the Stokes' number of the particle, i.e., the ratio of the particle time scale to the flow time scale. The time scale of a particle is $(\Delta\rho\, s^2)/(18\mu)$, where $\Delta\rho$ is the difference in density between the particle and the fluid and μ is the dynamic viscosity of sea water (McCave, 1984). The time scale of the flow can be calculated under dimensional grounds (Ozmidov, 1992) from empirical data. In the case of pelagic particles the Stokes' numbers are very small for the whole range of particle sizes (Fig. 6). For particles with very small Stokes' number the expression for the ratio between particle and fluid diffusivity is (Wang and Stock, 1993):

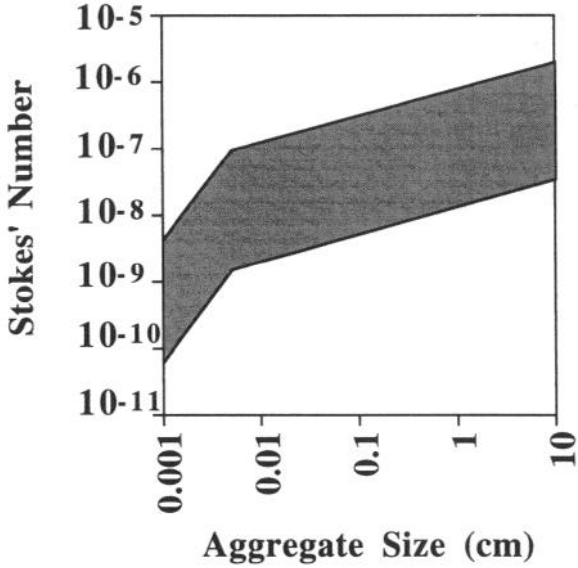

Figure 6. Size spectrum of the Stokes number for pelagic aggregates. The time scale of the flow was calculated from simple dimensional grounds from Brainerd and Gregg (1993) data. The characteristics of the particles that were necessary to find their time scale were obtained from the model proposed by Ruiz (1996).

$$\frac{K_{\text{particle}}}{K_{\text{fluid}}} = \left(1 + \frac{(0.356w)^2}{\overline{u'^2}}\right)^{-1/2} \tag{8}$$

The effect of particle size on $K_{\text{particle}}/K_{\text{fluid}}$ is shown in Figure 7. It is clear that the particle-to-fluid diffusivity ratio for a fixed particle size is strongly affected by the turbulence intensity due to the cyclic variation through the crossing trajectory effect.

e. Production and aggregation terms

Finally we must consider the fact that the advection-diffusion equation does not control the whole dynamics of particles in the mixed layer and that other terms added to the advection-diffusion equation might be important in determining the vertical distribution of particles in that layer. Among those terms are the generation (growth) of new particles because it might display vertical heterogeneity and the aggregation of particles because it is a nonlinear term.

i. Production. We investigated the effect, on the correction factor F, of including a source term that depends linearly on the concentration of particles, $rC(\eta, \tau)$, where r is the rate of growth. The important parameter is not, however, r but the dimensionless growth rate ($\phi = rh/w$) that results from the nondimensionalization of the advection-diffusion equation with a growth term. Two different tests were made to

Table 2. The advection-diffusion equation plus other terms. F_1, F_2 and F_3 correspond to the correction factor obtained from: only the advection and diffusion terms (F_1), the advection and diffusion terms plus a depth-dependent growth term (F_2) and the advection and diffusion plus an aggregation term (F_3).

s (cm)	w (m d^{-1})	h(m)	D	ϕ	rh^2/K	C (l^{-1})	$\omega Ch/w$	F_1	F_2	F_3
0.001	0.7	100	14.256	142.857	10	1000	0.062	1.012	0.720	1.012
0.01	21.8	100	0.459	4.587	10	100	0.043	1.386	1.097	1.385
0.1	55.0	100	0.182	1.818	10	10	0.368	2.041	1.798	2.013
1	137.0	100	0.073	0.730	10	1	3.187	3.805	3.654	3.344
0.001	0.7	50	28.571	71.429	2.5	1000	0.031	1.006	0.931	1.006
0.01	21.8	50	0.917	2.293	2.5	100	0.022	1.188	1.112	1.188
0.1	55.0	50	0.364	0.909	2.5	10	0.184	1.494	1.425	1.488
1	137.0	50	0.146	0.365	2.5	1	1.593	2.323	2.269	2.178
0.001	0.7	25	57.143	35.714	0.625	1000	0.016	1.003	0.984	1.003
0.01	21.8	25	1.835	1.147	0.625	100	0.011	1.092	1.073	1.092
0.1	55.0	25	0.727	0.455	0.625	10	0.092	1.239	1.220	1.238
1	137.0	25	0.292	0.182	0.625	1	0.797	1.624	1.608	1.591

examine the effect of including this term. The effect of including a ϕ independent of depth and with a high value of 20 (resulting, for instance, from particles settling to 5 m d^{-1} with a growth rate of 1 d^{-1} and in a mixed layer 100 m deep) was first considered. The inclusion of this growth rate produced a negligible effect on the correction factor F. Secondly, the effect that the inclusion of a growth term that decreases with depth has on F was investigated. A case that might happen when exponentially decaying light is limiting the biological production in part of the mixed layer. This exponentially decaying growth could affect the value of the correction factor as it accumulates particles in the surface. We, then, implemented a growth rate that decreases with depth, $\phi(\eta)$, according to:

$$\phi(\eta) = \phi_0 e^{-2.3\eta} \tag{9}$$

where ϕ_0 is the dimensionless growth rate at the surface (as calculated from $\phi_0 = rh/w$, with $r = 1$ d^{-1}). The decay rate applied (-2.3) corresponds to a decrement of one order of magnitude in the production of particles. This is within the range of variability of biological production observed in the mixed layer (Kirk, 1994). The results of this test are shown in Table 2. In this table, the deviation from the value of the correction factor that results from different combinations of settling velocities and mixed layer depths (the diffusivity was held constant at a standard value of 1000 m^2 d^{-1}) is represented. The high values of ϕ_0 appearing in Table 2 arise as combination of different settling velocities of particles and mixed layer depths, since the production of biological particles is unlikely to take values much higher than 1 d^{-1}. In this table we can see that for certain values of ϕ_0 the correction factor may change from the one resulting from the simple advection-diffusion equation. Significant deviations are present only when the dimensionless growth rate is higher than one, as expected from a nondimensionalized equation. High values of ϕ_0 are associated to low w and for low w one does not expect low D. Therefore, the deviation from the correction factor occurring because the inclusion of a growth term will mainly affect slow sinking particles where the correction factor is not so important. This feature is clear in Table 2 where it can be seen that the significant deviations from the correction factor are mainly associated with low values for the factor.

Another interesting feature of Table 1 is that values of the correction factor lower than one are possible when particle production in the mixed layer decreases with depth. In these cases, turbulence will increase the export of particles from the mixed layer via sedimentation. The reason for this is that, with low values of turbulence intensity, particles will accumulate in the upper part of the mixed layer and, therefore, the average concentration of particles in the mixed layer will be higher than the concentration at the bottom. In these cases, high values of turbulence intensity will make the concentration of particles more uniform; thus, increasing the concentration of particles at the bottom (and therefore also the sedimentation flux). This situation occurs when $rh/w > 1$ and $rh^2/K > 1$.

ii. Aggregation. The vertical distribution of particles in the mixed layer might also be affected by the aggregation of particles. This term is nonlinear since it depends on the square of the aggregate concentration. This nonlinear dependence on aggregate concentration may also alter the value of the correction factor as the loss of particles by aggregation will be higher where the concentration of particles is higher, that is, at the bottom of the mixed layer as we saw from Figure 1. To analyze this case we included an aggregation term in Eq. (2) of the form (Hill, 1992; Hill *et al.*, 1992):

$$\begin{aligned} \text{Aggregation} &= -\omega C^2(\eta, \tau) \\ \omega &= \alpha E \beta \\ \beta &= 1.08(2s)^{7/3}\epsilon^{1/3} \end{aligned} \tag{10}$$

where s is the diameter of the particle and $C(\eta, \tau)$ is in this case number concentration of particles. ϵ is the rate of dissipation of turbulent kinetic energy with values for the mixed layer of the order of 10^{-9} to 10^{-7} m^2 s^{-3} (Brainerd and Gregg, 1993). We chose an intermediate value of 10^{-8} m^2 s^{-3}. In Eq. (10), ω is an aggregation coefficient, β is the aggregation kernel by turbulent shear and E is the probability that two particles make contact once they are in close proximity. The value of E for particles of equal size is 0.83 (Hill, 1992). α is the sticking efficiency (the probability that two particles remain stuck after contact), its value varies between 0 and 1 depending on different factors such as the physiological conditions in the case of phytolankton cells or the presence of high concentrations of transparent exopolymer particles (Kiørboe *et al.*, 1990; Kiørboe and Hansen, 1993; Alldredge *et al.*, 1993; Passow *et al.*, 1994). Therefore, it is an important parameter determining when the aggregation term will significantly affect the vertical distribution of particles in the mixed layer. The value of 0.1 is commonly accepted in particle models (McCave, 1984; Jackson, 1990).

The approach for analyzing the effect of aggregation must be slightly different from which we have followed so far. The reason for this is that aggregation depends on the size and concentration of the particles considered. For that reason, when testing the effect of aggregation on the value of the correction factor, we have kept the concentrations of particles constant in the mixed layer according to the values given in Table 2. In Table 2 each particle is associated (because of its size) to a certain settling velocity, an aggregation coefficient and a concentration. The concentrations displayed in Table 2 intend to be only representative of the order of magnitude of the number concentration of particles of a certain size (Riebesell, 1992; Lampitt *et al.,* 1993a,b; Ruiz *et al.,* 1992; Ruiz, 1993). The aggregation coefficient ω has dimensions of (L^3T^{-1}) so that when nondimensionalizing the advection-diffusion equation (with the aggregation term) the resulting parameter ($\omega h/w$) has dimensions of L^3. A consequence of this is that the importance of the aggregation term does not depend only on ω, h and w but also on the concentration of particles. Therefore, the dimensionless parameter indicating deviations of the correction factor is $\omega Ch/w$. As evident from Table 2, when this parameter is higher than one the correction factor suffers significant deviations from the values of the simple advection-diffusion equation. This situation occurs more likely for large particles because of the increase in ω associated with large particles can, in certain situations, compensate (in the dimensionless group $\omega Ch/w$) for the low concentrations and high w of these particles.

For the sake of simplicity the analysis on the effect of aggregation presented above has been exemplified in a case in which all particles have a similar size. For aggregation of particles of different sizes, we need to analyze two different dimensionless groups, one for the loss and the other for the production of particles of that size. The loss of particles in the size class i due to aggregation would be:

$$\text{Loss by Aggregation} = C_i \sum_j \omega_{ij} C_j \tag{11}$$

where C_i is the concentration of particles in the size class in which the effect of turbulence on sedimentation loss is to be analyzed. ω_{ij} is the same as in Eq. (10) but for the collision of particles of different size (Hill, 1992). Then, the dimensionless number for the loss by aggregation is $h/w\ \Sigma_j\ \omega_{ij}C_j$. On the other hand, the generation of particles in the size class i by aggregation of smaller particles would be:

$$\text{Generation by Aggregation} = \frac{1}{2} \sum_{j+m=i} \omega_{jm} C_j C_m. \tag{12}$$

To analyze this case we must take into account that the continuum of particle sizes is usually divided into size classes in $\log_2$ increments of particle mass (Platt and Denman, 1977, 1978; Jackson and Lochmann, 1992; Hill, 1992). With this arrangement of size classes, the generation of a particle in size class i by aggregation of two particles of smaller size implies that at least one of the two particles must belong to the size class i-1 (Jackson and Lochmann, 1992). Thus, Eq. (12) can be rewritten as:

$$\frac{1}{2} C_{(i-1)} \sum_{j \leq i-1} \omega_{j(i-1)} C_j \tag{13}$$

and then the dimensionless number for the generation of particles by aggregation would be $h/2w \sum_{j \leq i-1} \omega_{j(i-1)} C_j$.

When either

$$\frac{h}{w} \sum_j \omega_{ij} C_j \text{ or } \frac{h}{2w} \sum_{j \leq i-1} \omega_{j(i-1)} C_j$$

are higher than one, the aggregation of particles will affect the value of F and the approach presented in this paper is not reliable.

3. Discussion

Turbulence is one of the main processes affecting ecosystem dynamics in the upper layers of the ocean. The analysis presented above parameterizes its effect on the sedimentation of pelagic particles in a simple way that allows its use in ecosystem models. For that, we only need to calculate the ratio (F) of particle concentration at the bottom of the mixed layer to the average concentration in that layer. This ratio depends on turbulence levels and can be used as a correction factor for the sedimentation term usually implemented in ecosystem models (Eq. (7)). When turbulence levels are high ($D > 1$), F is close to 1 and the use of a correction factor is not necessary. However, when turbulence levels are low ($D < 1$) the analysis presented above demonstrates that F may be much higher than 1 for a wide size range of pelagic aggregates. In these cases, the use of a noncorrected sedimentation term implies a severe underestimation of the flux.

Although the analysis made in this paper is based on pelagic aggregates, the approach is equally valid when studying other pelagic particles such as phytoplankton cells. Thus, values of D lower than 1 can be expected for some phytoplankton species when considering their sedimentation velocities (Smayda, 1970). This has identical consequences for the sedimentation of phytoplankton cells as for the sedimentation of aggregates.

The approach presented in this paper is only a partial solution to the problem of modeling the role of turbulence in particle sedimentation. One of the reasons for this is that the distribution of particles within the mixed layer takes approximately two dimensionless times to adjust to certain turbulence levels. This dimensionless time implies dimensional times of the order of hours-days for the particles in which the correction factor is important. Thus, those models that assume either a constant turbulence level in the mixed layer or a turbulence that varies on a time scale longer than days can directly use the asymptotic value of the factor calculated in this paper. For models in which turbulence is considered to vary on a time scale of days the factor gives an estimate of the flux underestimation.

For models in which turbulence is considered to vary in a daily cycle, it is still possible to calculate the oscillating factor by considering the dimensionless frequency of oscillation; as it was made in Figure 5. According to Ruiz (1996), the presence of daily cycles of turbulence is the process more likely to produce the daily cycles of marine snow discovered by Lampitt *et al.* (1993a). The analysis presented above supports this conclusion as it displays the existence of two processes that follow the daily cycle of turbulence in the mixed layer and that affect the sedimentation of aggregates. Thus, daily cycles of fluid eddy diffusivity will produce daily cycles of the correction factor (F). Also, daily cycles of fluid turbulent velocities will make the Yudine effect influence cyclically to the aggregate diffusivity.

These two processes will not affect all the aggregates in the same way but their effect will be more important for large aggregates than for small ones. Thus, large aggregates have low dimensionless frequencies of oscillation because of their high settling velocities. Consequently, their correction factors for sedimentation has wider oscillating amplitudes than those of small aggregates (see Fig. 5). Also, the daily reduction in the diffusivities of particles associated with the Yudine effect will affect large aggregates more (see Fig 7). Finally, the correction factor changes with D in a nonlinear way and, when the value of D is low, small changes in D produce big changes in the factor. Since the value of D is low for large particles (see Fig 4), their flux will be more sensitive to changes in D than the flux of small ones.

The influence of terms different from advection and diffusion in the vertical distribution of particles also limits the applicability of the approach developed. We have tested two terms that are important when considering the vertical distribution of biological particles: a growth and an aggregation term. The growth term has no important effect on the value of the correction factor. However, the nonlinear characteristic of the aggregation term affects in some cases the value of the correction factor and invalidates the approach presented above. Nevertheless, this case can be detected in terms of dimensionless numbers.

An important consequence of the approach developed in this paper affects aggregation theory itself as it demonstrates the importance of resolving the vertical scale in the modeling approaches resulting from this theory. The reasons for this are clear when looking at Figure 1. In the aggregation models so far implemented to study the dynamics of pelagic particles, it is considered that their size spectrum is controlled by aggregation and break-up processes and that there is no vertical variability. However, the size distribution of particles at a certain depth will not be controlled uniquely by these processes since the advection and diffusion terms must

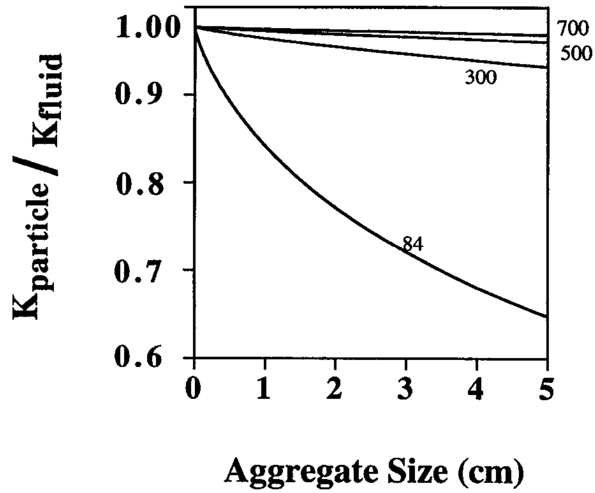

Figure 7. Size spectrum of $K_{\text{particle}}/K_{\text{fluid}}$ under different conditions of turbulence. The numbers in the curves are the magnitude of turbulence velocities (md^{-1}) as determined from simple dimensional grounds (Ozmidov, 1992) from Brainerd and Gregg (1993) data. Values as low as 83 m d^{-1} may be reached during day whereas during the night values of 700 m d^{-1} and higher are reached.

be taken into account. These terms generate a different distribution of the particles throughout the mixed layer depending on their size: large aggregates tend to accumulate at the bottom whereas small aggregates will be more uniformly distributed. This feature has been reported in the mixed layer by Alldredge and Gotschalk (1989) who described how the size of the aggregates measured in the mixed layer increased with depth. Thus, the theoretical results of this paper show the importance of considering the advection and diffusion terms when studying the dynamics and the export flux of particulated matter from the mixed layer. The inclusion of these terms in future models of particle dynamics will increase our understanding on the flux to the deep ocean of the particulate matter resulting from oceanic photosynthesis.

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APPENDIX 1

The boundary value problem (5) was solved through the transformation given by Jost (1960):

$$C = C^* e^{(\eta/2D - \tau/4D)} \tag{A.1}$$

and, then, separation of variables to obtain an analytical solution for C:

$$C(\eta, \tau) = e^{(\eta/2D - \tau/4D)} \sum_{0}^{\infty} a_k e^{-\lambda_k D \tau} M_k(\eta) \tag{A.2}$$

where the M_k are:

$$M_k = D \cos(\sqrt{\lambda_k}\eta) + \frac{\sin(\sqrt{\lambda_k}\eta)}{2\sqrt{\lambda_k}} \tag{A.3}$$

and the λ_k are the roots of the transcendental equation:

$$2\sqrt{\lambda}D \cos(\sqrt{\lambda}) + \frac{\sin(\sqrt{\lambda})}{2} - 2D^2\lambda \sin(\sqrt{\lambda}) = 0. \tag{A.4}$$

The coefficients a_k of Eq. (A.2) depend on the initial distribution of particles, $f(\eta)$. They were obtained, by applying orthogonal functions theory, as:

$$a_k = \frac{\int_0^1 e^{(-\eta/2D)} f(\eta) M_k(\eta) d\eta}{\int_0^1 M_k(\eta) M_k(\eta) d\eta} \tag{A.5}$$

When the initial distribution of particles in the mixed layer is uniform, $f(\eta) = A$, the coefficients a_k are:

$$\begin{aligned}
a_k &= \frac{n_1 + n_2}{d_1 + d_2 + d_3} \\
n_1 &= \left(\frac{AD}{\left(-\frac{1}{2D}\right)^2 + \lambda_k}\right)\left\{\left[\left(-\frac{1}{2D}\right)\cos(\sqrt{\lambda_k}) + \sqrt{\lambda_k}\sin(\sqrt{\lambda_k})\right]e^{(-1/2D)} + \left(\frac{1}{2D}\right)\right\} \\
n_2 &= \left(\frac{A}{2\sqrt{\lambda_k}\left(\left(-\frac{1}{2D}\right)^2 + \lambda_k\right)}\right)\left\{\left[\left(-\frac{1}{2D}\right)\sin(\sqrt{\lambda_k}) - \sqrt{\lambda_k}\cos(\sqrt{\lambda_k})\right]e^{(-1/2D)} + \sqrt{\lambda_k}\right\} \\
d_1 &= D^2\left\{\frac{1}{2} + \frac{\sin(2\sqrt{\lambda_k})}{4\sqrt{\lambda_k}}\right\} \\
d_2 &= \frac{1}{4\lambda_k}\left\{\frac{1}{2} - \frac{\sin(2\sqrt{\lambda_k})}{4\sqrt{\lambda_k}}\right\} \\
d_3 &= \frac{D}{\sqrt{\lambda_k}}\left\{-\frac{\cos(2\sqrt{\lambda_k}) - 1}{4\sqrt{\lambda_k}}\right\}
\end{aligned} \tag{A.6}$$

APPENDIX 2

Analytical expression for the integral over the mixed layer of the solution to the boundary value problem

$$A(\tau) = e^{(-\tau/4D)} \sum_{0}^{\infty} a_k e^{-\lambda_k D \tau}[s_1 + s_2]$$

$$s_1 = \frac{D\left\{\left[\frac{\cos(\sqrt{\lambda_k})}{2D} + \sqrt{\lambda_k}\sin(\sqrt{\lambda_k})\right]e^{(1/2D)} - \left(\frac{1}{2D}\right)\right\}}{\left(\frac{1}{2D}\right)^2 + \lambda_k} \tag{A.7}$$

$$s_2 = \frac{\left[\frac{\sin(\sqrt{\lambda_k})}{2D} - \sqrt{\lambda_k}\cos(\sqrt{\lambda_k})\right]e^{(1/2D)} + \sqrt{\lambda_k}}{2\sqrt{\lambda_k}\left(\left(\frac{1}{2D}\right)^2 + \lambda_k\right)}$$

Notation*

$C(z, t)$	L^{-3}	Concentration of aggregates.
$\overline{C}(t)$	L^{-3}	Average concentration of aggregates in the mixed layer.
$A(\tau)$	L^{-2}	Concentration of aggregates, per unit area, in the mixed layer.
$\psi(\eta, \tau)$	L^{-1}	Shape function for the distribution of aggregates in the mixed layer.
w	LT^{-1}	Sinking velocity of the particle.
z	L	Depth.
t	T	Time.
K	L^2T^{-1}	Diffusivity.
D		Dimensionless diffusivity.
τ		Dimensionless time.
η		Dimensionless depth.
F		Correction factor.
$\overline{u'^2}$	L^2T^{-2}	Mean square of the turbulent fluctuating velocity.
r	T^{-1}	Dimensional growth rate.
ϕ		Dimensionless growth rate.
ω	L^3T^{-1}	Aggregation coefficient.
s	L	Diameter of the particle.
ϵ	L^2T^{-3}	Rate of dissipation of turbulent kinetic energy.
μ	$ML^{-1}T^{-1}$	Sea water dynamic viscosity.

*The second column gives the dimensions of the variable in the first column.

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