# A simple model for the break-up of marine aggregates by turbulent shear

Marine snow
Aggregate break-up
Pelagic particle dynamics
Seston modelling

Neige marine
Rupture des agrégats
Dynamique des particules pélagiques
Modèle de seston

**Javier RUIZ** [a] **and Alfredo IZQUIERDO** [b]

[a] Departamento de Biologia y Ecologia, Facultad de Ciencias del Mar, Universidad de Cádiz, 11510 Puerto Real, Cádiz, Spain.
E-mail: javier.ruiz@uca.es

[b] Departamento de Física Aplicada, Universidad de Cádiz, Spain.

Received 16/08/95, in revised form 07/02/96, accepted 13/08/96.

ABSTRACT

A simple model for the break-up, by turbulent shear, of marine aggregates is proposed. Three factors are considered to control the rupture of aggregates of different sizes. First, turbulence exerts stronger shear forces on bigger aggregates. Second, big aggregates are weaker than small ones because their porosity increases. Third, the rate of rupture of an aggregate is considered to be dependent on the point in the aggregate where the rupture takes place. This third factor allows the determination of the size distribution resulting from aggregate break-up. By considering these factors the model predicts the break-up rates of aggregates of different sizes and the aggregates' size- distribution resulting from this break-up. Therefore, the model represents an attempt to complete the general dynamic equation presented by McCave (1984) as the governing equation for the dynamics of pelagic particles. When implemented in a more general model of particle dynamics, the break-up model produces results that are consistent with empirical observations of the break-up of aggregates. Also, it provides a theoretical explanation for these results.

RÉSUMÉ

Modèle simple pour la rupture des agrégats marins par cisaillement turbulent.

On propose un modèle simple pour la rupture de la neige marine par cisaillement turbulent. Le modèle envisage trois procédés qui contrôlent la rupture des agrégats de différentes tailles. Premièrement, la force de cisaillement exercée par l'action de la turbulence est plus forte sur les grands agrégats que sur les plus petits. Deuxièmement, les grands agrégats sont moins résistants que les plus petits parce que la porosité augmente avec la taille de l'agrégat. Troisièmement, le taux de rupture dépend du point où a lieu la rupture, à l'intérieur de l'agrégat. Ce troisième procédé permet de déterminer la répartition des tailles qui résulte de la rupture. Grâce à ces procédés, le modèle est capable de prévoir les taux de rupture des agrégats de différentes tailles, aussi bien que la répartition des tailles des particules résultantes. Par conséquent, le modèle représente une tentative pour compléter l'équation générale de la dynamique des particules pélagiques présentée par McCave (1984). Lorsqu'il est mis en œuvre dans un modèle plus général de dynamique des particules, le modèle de rupture donne des résultats qui sont en bon accord avec les observations empiriques de rupture des agrégats. De plus, le modèle fournit l'explication théorique de ces observations.

*Oceanologica Acta*, 1997, **20**, 4, 597-605.

## Notation

| | |
|---|---|
| $c_3, c_4$ | Constants of Jackson (1995) break-up model. |
| $d$ | Aggregate diameter (cm). |
| $f$ | Force per unit area exerted by the shear flow on aggregate surface (dyn $cm^{-2}$). |
| $F_a$ | Binding force (dyn). |
| $F_b$ | Shear force exerted on the aggregate by shear flow (dyn). |
| $K$ | Factor transforming the balance between the break and binding forces into a break-up rate ($cm^{-1}$ $s^{-1}$). |
| $m(d, x)$ | Mass lost by an aggregate of size $d$ due to a fracture at distance $x$ from its centre (nanomoles of $N$). |
| $M(d)$ | Mass of an aggregate of size $d$ (nanomoles of $N$). |
| $M_i$ | Nominal mass of aggregates in size class $i$ (nanomoles of $N$). |
| $n(d)$ | Function describing the size distribution of particles ($cm^{-1}$). |
| $p(d, x)$ | Density function describing the break-up rate of aggregates ($cm^{-1}$). |
| $P$ | Aggregate porosity. |
| $Q_i$ | Mass concentration of aggregates in size class $i$ ($\mu$M of $N$). |
| $r_i$ | Decay rate of particles in size class $i$ due to break-up ($s^{-1}$). |
| $R(d)$ | Break-up rate of an aggregate of size $d$ ($s^{-1}$). |
| $S$ | Constituent sectional area ($cm^2$). |
| $t_{ij}$ | Rate of production of $i$-aggregates due to the break-up of $j$-aggregates ($s^{-1}$). |
| $u(d, x)$ | Volume lost by an aggregate of size $d$ due to a fracture at distance $x$ from its centre ($cm^3$). |
| $V(d)$ | Volume of an aggregate of size $d$ ($cm^3$). |
| $V_c$ | Volume occupied by the constituent matter of an aggregate ($cm^3$). |
| $x$ | Distance from the centre of the aggregate (cm). |
| $\varepsilon$ | Rate of turbulent kinetic energy dissipation ($cm^2$ $s^{-3}$). |
| $\delta_{k1}\, d_{k2}$ | |
| $\delta_{g1}\, d_{g2}$ | Conditional functions for break-up. |
| $\mu$ | Sea-water dynamic viscosity (gr $cm^{-1}$ $s^{-1}$). |
| $\rho_c$ | Constituent matter density (g $cm^{-3}$). |
| $\phi$ | Proportion of nitrogen in constituent matter. |
| $\sigma$ | Factor defining how strong the particles of an aggregate are bound to each other (dyn $cm^{-2}$). |
| $v$ | Sea-water kinematic viscosity ($cm^2$ $s^{-1}$). |

## INTRODUCTION

The formation of algal aggregates is a characteristic of many bloom situations. They result from phytoplankton aggregation; because their size is greater than that of the cells forming them and because large particles sink faster than small ones, aggregation causes an increase in the rate at which phytoplankton are lost from surface waters. The effect of break-up is opposite to that of aggregation. The transport of mass from large particles toward small ones decreases the rate of loss of phytoplankton from surface waters. The rate at which phytoplankton sink out of the euphotic zone will consequently depend on both processes: aggregation and break-up.

Due to the importance of aggregation in the vertical transport of matter, its study has received much attention. Thus, the characterization of the aggregation process has significantly advanced through empirical and modelling work. The importance of aggregation in the dynamics of marine particles is now clear as a result of field (Krank and Milligan, 1988; Alldredge and Gotshalk, 1989; Riebesell, 1992) and laboratory (Krank and Milligan, 1980) observations. The basic mechanisms controlling the interaction of marine particles have been identified (McCave, 1984; Hill, 1992; Delichatsios and Probstein, 1975) and several authors have successfully implemented these mechanisms to reproduce the dynamics of particulate matter during blooms (Jackson, 1990; Jackson and Lochmann, 1992; Hill, 1992; Riebesell and Wolf-Gladrow, 1992). These models are able to predict the formation of particles of large size and high sinking speed from phytoplankton cells during bloom events.

The study of aggregate break-up, on on the other hand, has received much less attention although it has the same potential importance as aggregation in controlling the final fate of particulate matter. Empirical evidence seems contradictory when evaluating the importance of break-up in particle dynamics. Laboratory experiments (Alldredge *et al.*, 1990) indicate that marine snow aggregates are strong enough to resist the turbulence levels of ocean surface waters, and consequently that the break-up of marine aggregates may not be an important process controlling the dynamics of marine aggregates. However, field observations (Riebesell, 1992) show how the break-up of marine aggregates is a common feature in surface waters when turbulence levels are sufficiently high. This contradiction is apparently related to the greater size of the aggregates studied *in situ* in comparison with those studied in the laboratory (Riebesell, 1992). On the other hand, theoretical investigations of aggregate break-up have made it possible to estimate the maximum size of stable aggregates by means of dimensional analysis (Tomi and Bagster, 1978; Tambo and Hozumi, 1979; Hunt, 1986) or modelling of the flow around porous spheres (Adler and Mills, 1979). Nevertheless, no model of aggregate break-up as yet exists that is able to predict not only the maximum size of stable aggregates but also the size distribution of the detached particles.

There are several particle models which incorporate the break-up of marine aggregates in the investigation of the dynamics of particulate matter. Riebesell and Wolf-Gladrow (1992) include a break-up term in their model, but this term only accounts for biological break-up, *i.e.* considers the separation of individual cells from a chain of cells as the only mechanism of transfer of mass towards smaller particles. Hill (1992) includes a simple break-up term in his model of particle dynamics. He assumes that shear cannot disrupt particles smaller than a maximum size but inhibits the formation of particles larger than that maximum size. Therefore, the break-up terms included up to now in particulate matter models lack the detail necessary to reproduce the main factors affecting the physical break-up of aggregates. They also fail in estimating the redistribution of mass along the size spectrum of marine particles as a result of break-up. Since break-up is one of the main processes affecting the dynamics of marine particles, a model for this process is a necessary step for the reliable modelling of particulate matter. In this paper we propose a very simple model for the break-up of algal aggregates, based on simple geometric and dimensional considerations. The model produces results consistent with field and laboratory observations of the effect of turbulence on the break-up of marine aggregates.

## MODEL

### Origin of the model

Due to the characteristics of turbulent flows, as well as the heterogeneity of aggregate composition and structure, the rupture process of an aggregate population cannot be treated in a deterministic fashion. Instead, we follow a probabilistic approach similar to that of Pandya and Spielman (1982), and consider the break-up process as the product of a rupture frequency and a probability density function. In the modelling process, it will be subsequently assumed that this product is proportional to the balance between the shear force generated by turbulence and the force that binds together the particles in the aggregate. On this point, the approach is akin to that suggested by different authors working on waste-water treatment as a means of determining the maximum aggregate size in an agitated suspension (Tomi and Bagster, 1978; Tambo and Hozumi, 1979; Hunt, 1986).

According to the Kolmogorov assumption of isotropic turbulence, the magnitude of the shear force per unit area exerted by water on an aggregate surface ($f$) can be derived by dimensional analysis as $f \propto \mu\,(\varepsilon/v)^{1/2}$. Where $\mu$ is the dynamic viscosity, $v$ is the kinematic viscosity and $\varepsilon$ is the rate of viscous dissipation of turbulent kinetic energy. The shear force exerted on the aggregate by the shear flow ($F_b$), responsible for the rupture of the aggregates, will be the result of integrating $f$ over the surface of the aggregate. $F_b$ will be highest in the middle of the aggregate, where the integrated $f$ on either side of the aggregate is maximum, whereas closer to the pole of the aggregate the integrated force will be smaller, since there is less particle area. The shear force at a certain distance, $x$, from the centre of the aggregate will then be:

$$F_b \propto \mu\,(\varepsilon/v)^{\frac{1}{2}} \left(\frac{d^2}{2} - dx\right) \tag{1}$$

On the other hand, the break-up rate must also depend on the forces keeping the aggregate bound, the binding force ($F_a$). This force is proportional to the constituent sectional area ($S$), *i.e.* the area of constituent matter in a section of the aggregate.

$$F_a = \sigma\,S \tag{2}$$

where $\sigma$ is a factor with dimensions of force per unit area. It is a measure of the strength with which the particles of an aggregate are bound to each other.

If the fracture is across the equator of the aggregate, $S$ is proportional to $V_c^{2/3}$; where $V_c$ is the volume occupied by constituent matter. In an aggregate of diaméter $d$, $V_c$ is:

$$V_c = (1 - P)\,(\pi/6)\,d^3 \tag{3}$$

where $P$ is the porosity of the aggregate. Aggregate porosity is an increasing function of aggregate size (Tambo and Watanabe, 1979; Alldredge and Gotschalk, 1988). Alldredge and Gotschalk (1988) developed an empirical formula relating the porosity of an aggregate to its size that will be used in this paper. This formula is:

$$(1 - P) = 8 \times 10^{-3}\,d^{-1.6} \tag{4}$$

where the diameter is in millimetres. Consequently, $S$, in the equatorial section of an aggregate, is:

$$S \propto V_c^{2/3} \propto d^2\,(1 - P)^{2/3} \tag{5}$$

However, if the fracture occurs through a section other than the equatorial one, $S$ is:

$$S \propto ((d/2)^2 - x^2)\,(1 - P)^{2/3} \tag{6}$$

At this point, we have analytic expressions for the two forces controlling the break-up of aggregates: the binding force and the shear force. The balance between these two forces ($F_b/F_a$) is considered to control the maximum size of stable aggregates (Tomi and Bagster, 1978; Tambo and Hozumi, 1979; Hunt, 1986). It is possible that, in addition to the maximum aggregate size, this balance also control the process of aggregate rupture. This process can be represented as the product of the rupture frequency of an aggregate, $R(d)$, and a probability density function (Pandya and Spielman, 1982). We will name $p(d, x)$ to the probability density function; $p(d, x)\,dx$ equals the probability of a rupture whose distance from the aggregate centre lies within the radius interval $(x, x + dx)$. Hence, aggregate rupture is formulated as:

$$R(d)\,p(d, x) = K\,\frac{F_b}{F_a} = K\,\frac{\pi\mu\,(\varepsilon/v)^{\frac{1}{2}}\,(d^2/2 - xd)}{\sigma\,((d/2)^2 - x^2)\,(1 - P)^{\frac{2}{3}}} \tag{7}$$

where $K$ is a constant with dimensions of $T^{-1}\,L^{-1}$.

To calculate the rate at which aggregates of a certain size lose mass due to break-up, we must also compute the amount of mass lost in each fracture. The next expression calculates that mass:

$$m(d,\,x) = \frac{u(d,\,x)}{V(d)}\,M(d) = \left(\frac{d-3\,x}{2\,d} + \frac{2\,x^3}{d^3}\right) M(d) \qquad (8)$$

where $m(d,\,x)$ and $u(d,\,x)$ are respectively the mass and volume lost by an aggregate of size d due to a fracture at a distance $x$ from its centre, $V(d)$ is the volume of the aggregate and $M(d)$ its mass. We have expressed the mass of aggregates in this paper as nanomoles of nitrogen, since we test below the break-up model by implementing it in a more general nitrogen-based model for particle dynamics.

The product $R(d)\,p(d,\,x)\,m(d,\,x)$ versus aggregate radius is represented in Figure 1 for several aggregate diameters. The Figure shows how break-up rates of large aggregates ($e.g.\ d = 2$ cm) are higher than the rates of smaller ones ($e.g.\ d = 200\,\mu$m). Figure 1 also shows how the main loss of mass due to break-up is due not to surface fractures, but to those which do not occur close to the surface of the aggregates. This is because of the small mass which is lost by an aggregate when a fracture occurs at $x \sim d/2$, since then $m(d,\,x)$ is small.

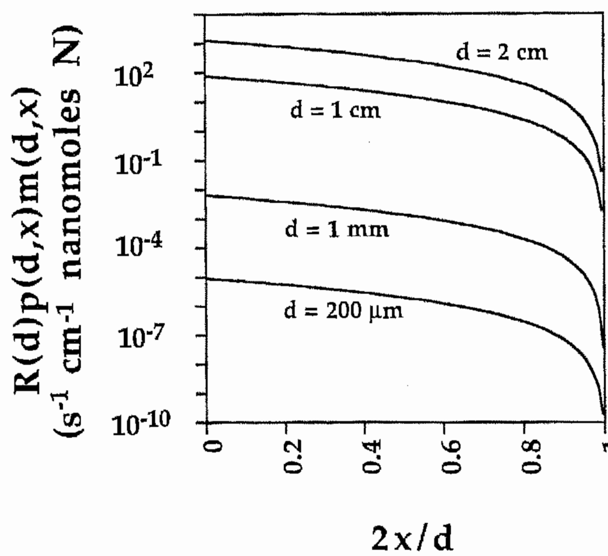


Figure 1

*Values of the product R(d)p(d, x)m(d, x) versus particle radius for several particle diameters.*

With the equations already developed, we can calculate the rate of mass loss by aggregates in a certain size range $(d,\ d+\Delta d)$. This rate depends on the concentration of particles in that size and also on the size distribution of the particles (as the functions described above depend on the size of aggregates). These features are usually represented by a function, $n(\eta)$, which, when integrated over a size interval $(n,\ \eta+\delta n)$, gives the abundance of particles in that size interval (*see* McCave, 1984 or Bader, 1970). The rate at which a population of aggregates in the size range $(d,\ d+\Delta d)$ loses mass can, therefore, be calculated by multiplying the abundance of aggregates of a certain size (given by $n(\eta)$) by the rate at which aggregates of that size lose mass and integrating over the size range $(d,\ d+\Delta d)$. The following expression performs that task:

$$\int_d^{d+\Delta d} n(\eta) \times \left\{ \left[\int_0^{\eta/2} \delta_{k1}(\eta,\,x)\,m(\eta,\,x)\,R(\eta)\,p(\eta,\,x)\,dx\right] + \left[\int_0^{\eta/2} \delta_{k2}(\eta,\,x)\,(M(\eta)-m(\eta,\,x))\,R(\eta)\,p(\eta,\,x)\,dx\right] \right\} d\eta \qquad (9)$$

where

$\delta_{k1}(\eta,\,x) = 1$ if $m(\eta,\,x) < M(d)$ or 0 if $m(\eta,\,x) \geq M(d)$

$\delta_{k2}(\eta,\,x) = 1$ if $M(\eta) - m(\eta,\,x) < M(d)$ or 0 if $M(\eta) - m(\eta,\,x) < M(d)$

The integrals within braces in eq. [9] calculate the rate at which aggregates lose mass due to break-up. There are two integrals because we must consider two ways in which mass in a certain size range can be lost after a fracture occurs. The first of these is represented by the first integral within braces. This integral accounts for the mass loss due to the separation of a portion $m(\eta,\,x)$ of aggregate, after a fracture at a distance $x$ from the centre of the aggregate has occurred. The portion detached from the aggregate, $m(\eta,\,x)$, can be of a size that lies within the size range studied $(d,\ d+\Delta d)$. In this case, despite the occurrence of a fracture, there is no net loss of mass from the size range. It can also happen that the detached portion is not in the size range studied, $m(\eta,\,x) < M(d)$. Only in this case will the break-up of an aggregate imply a net loss of mass from the size range studied. We must only compute those fractures in which a net loss of mass in the size range studied occurs. For that reason, we need to multiply that term by the function $\delta_{k1}(\eta, x)$. This function is equal to one when $m(\eta, x) < M(d)$ and zero when $m(\eta, x) \geq M(d)$. Consequently, with the inclusion of $\delta_{k1}(\eta,\,x)$, we achieve the aim of only considering fractures bearing a net loss of mass in the size range studied.

The second way of losing mass by break-up is represented by the second integral within braces of expression [9]. This integral accounts for the mass loss when the remnant part of the aggregate, after a fracture has occurred at $x$, has a size that does not lies within the size range studied. In this case, the mass lost after the fracture of an aggregate is not only $m(\eta,\,x)$ but the whole mass of the aggregate, $M(\eta)$. This is the result of both portions of the aggregate, $m(\eta,\,x)$ and $M(h) - m(\eta,\,x)$, being lost from the size range $(d,\ d+\Delta d)$. For this reason we must include a second integral in expression [9]. As with regard to the first way of losing mass, we need to include the function

$\delta_{k2}(\eta, x)$ whose value is one when there is a loss of mass in the size range. The value of $\delta_{k2}(\eta, x)$ is zero when the remnant aggregate is within the size range studied and, therefore, there is no net loss of mass.

The rate at which aggregates in a certain size range gain mass due to the break-up of larger aggregates is:

$$\int_{d+\Delta d}^{d_{max}} n(\eta) \times \left\{ \begin{array}{l} \left[\int_0^{\eta/2} \delta_{g1}(\eta, x)\, m(\eta, x)\, R(\eta)\, p(\eta, x)\, dx\right] \\ + \left[\int_0^{\eta/2} \delta_{g2}(\eta, x) \right. \\ \left. \quad \times (M(\eta) - m(\eta, x))\, R(\eta)\, p(\eta, x)\, dx\right] \end{array} \right\} d(\eta) \tag{10}$$

where

$\delta_{g1}(\eta, x) = 1$ if $M(d) \le m(\eta, x) < M(d + \Delta d)$ or 0 otherwise. And,

$\delta_{g2}(\eta, x) = 1$ if $M(d) \le M(\eta) - m(\eta, x) < M(d + \Delta d)$ or 0 otherwise.

The interpretation of the first and second integral of expression [10] is similar to that given in expression [9] but considering in this case the gain, rather than the loss, of mass in a size range.

When the model to implement these equations arranges the aggregates in size classes, as is the case in particle models, an expression of the type presented in eq. [11] must be obtained from equations [9] and [10]. Thus, for the different size classes the break-up rate is:

$$\frac{dQ_i}{dt} = -r_i Q_i + \sum_{j=i+1}^{j=max} t_{ij} Q_j \tag{11}$$

where $Q_i$ is the mass concentration of class $i$. $r_i$ represents the decay rate in the concentration of particles in size class i due to break-up and $t_{ij}$ represents the rate of production of i-aggregates due to the rupture of $j$-aggregates. A simplification in the formulation of $r_i$ and $t_{ij}$ can be achieved if it is assumed that the mass concentration, $n(d)$ $M(d)$, is constant within a size class. In this case, $r_i$ and $t_{ij}$ can be represented as:

$$r_i = \int_{d_{i-1}}^{d_i} \frac{R(\eta)}{(d_i - d_{i-1})\, M(\eta)} \times \left\{ \int_0^{\eta/2} p(\eta, x) \left[ (\delta_{k1}(\eta, x)\, m(\eta, x)) + \delta_{k2}(\eta, x)\, (M(\eta) - m(\eta, x)) \right] dx \right\} d\eta \tag{12}$$

$$t_{ij} = \int_{d_{j-1}}^{d_j} \frac{R(\eta)}{M(\eta)(d_j - d_{j-1})} \times \left\{ \int_0^{\eta/2} p(\eta, x) \left[ \delta_{g1}(\eta, x)\, m(\eta, x) + \delta_{g2}(\eta, x)\, (M(\eta) - m(\eta, x)) \right] dx \right\} d\eta$$

where the limits of size class $i$, $M(d_{i-1})$ and $M(d_i)$, are used for the conditional functions.

In this paper, we obtained the coefficients $t_{ij}$ and $r_i$ by a simple integration of the integrals within braces of eq. (10) through the Simpson rule. For each of the terms of this rule, the conditional expressions of eq. (10) were checked (for all the size classes smaller than the size class over which the integration is performed). When the condition was fulfilled, that term was added to that size class. In this way, an upper triangular matrix for the transfer of mass due to break-up is obtained (the size class over which the integration is being performed is the column, while the size classes receiving the mass are the rows). The diagonal elements of this matrix (which would be the $r_i$ terms) are zero, but they result from summing the values in that column and multiplying the result by $-1$ .

### The factor $K/\sigma$

In the above equations, there are two unknown factors: $\sigma$ and $K$. The probabilistic approach followed, only requires an estimation of the factor $K/\sigma$. This was made by using the fact that the size of the largest marine aggregates is usually in the range of millimetres to centimetres, and only very rarely is observed to be larger (Shanks and Trent, 1980; Alldredge and Gotshalk, 1989; Riebesell, 1992; Lampitt *et al.*, 1993). With this consideration, the value of $K/\sigma$ must be low enough to permit the formation of these aggregates but high enough to avoid the formation of larger ones. We made this estimation by including the break-up model in a more general nitrogen-based model of particle dynamics, and found that the value fulfilling the above conditions was of the order of 0.1 ($cm^{-1}$ $s^{-1}$)/(dyn $cm^{-2}$) .

## RESULTS

Figure 2 displays the result of a numerical experiment designed to show how the break-up process transfers mass along the size spectrum. In the experiment, the same initial concentration of 10 $\mu$M of nitrogen is located in different size classes in which the upper bound of each size class is twice the mass of the lower bound. For the lower bound of the smallest size class, the nominal mass of a solitary alga of 20 $\mu$m of equivalent spherical diameter was used. The nominal diameters corresponding to these size classes were obtained from the following equation:

$$M_i = \phi \rho_c (1 - P)(\pi d_i^3/6) \tag{13}$$

where $\rho_c$ is the density of the constituent matter of aggregates (1.17 g $cm^{-3}$) and $\phi$ is the proportion of nitrogen in constituent matter (a value of 0.01was chosen). After the calculation of $M_i$, the mass values were transformed from grams to nanomoles of nitrogen by using the adequate conversion factors. With this arrangement of size classes, the break-up model acts for one minute and the resulting size distribution of particles is recorded. Thus, the $x$-axis of Figure 2 represents the different size classes with

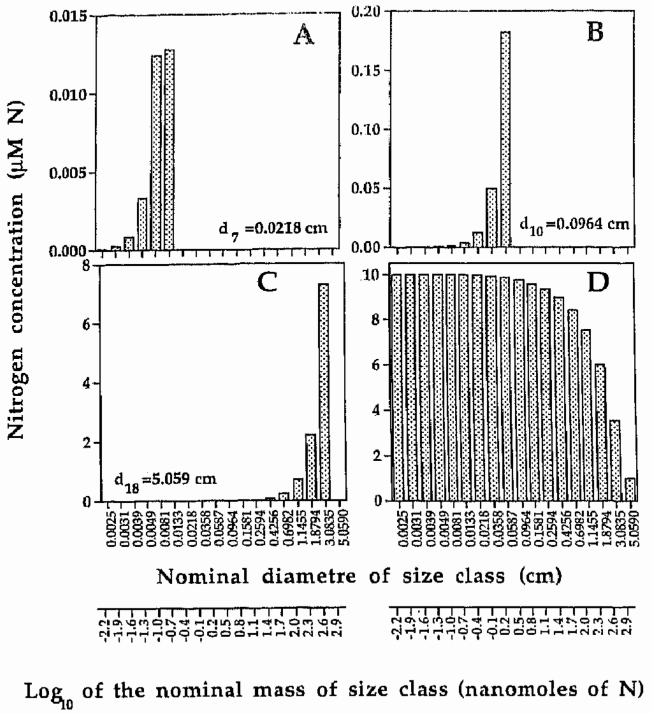


Figure 2

*Transfer of mass among different size classes generated by the break-up of aggregates. The figure exemplifies the process in three different size classes. Classes 7, 10 and 18 were chosen to represent the process for aggregates of different sizes. For instance, in Figure 2b, an initial concentration of 10 µM was set for size class 10 (the initial nitrogen of the other size classes was 0). After one minute of break-up, this size class produced a concentration of 0.18 µM in class 9, of 0.05 µM in class 8 and the shown concentrations for the other size classes. The same experiment was done for all the size classes and the nitrogen that remains in the same size class after the break-up is shown in Figure 2d. In the experiment, the values for* $\varepsilon$ *and* $K/\sigma$ *were* $3 \times 10^{-5}$ $cm^2\ s^{-3}$ *and 0.1* $(cm^{-1}\ s^{-1})/(dyn\ cm^{-2})$*, respectively.*

their respective nominal diametres and masses. The $y$-axis represents the concentration of nitrogen in the different size classes after break-up. In Figures $a$, $b$, $c$, this concentration is the result of the break-up of the 10 µM that were allocated (at the beginning of the experiment) in size classes 7, 10 and 18 respectively. It is clear that as a result of break-up there is no production of particles in size classes larger than those in which the initial concentration of nitrogen was allocated. For example, in Figure 2$b$, the 10 µM of nitrogen were allocated in size class 10 (nominal diameter of the class is 964 µm.) and, as a result of break-up, it transferred nitrogen to classes from 1 to 9. In Figure 2$d$, the $y$-axis represents the nitrogen that still remains in the same size class after one minute of break-up.

Two features are clear from Figure 2. First, the rate at which aggregates lose mass due to break-up increases strongly with the size of the aggregates. Second, the transfer of mass after the break-up of aggregates is not towards the basic particles forming the broken aggregates but towards particles of similar size, in coincidence with the empirical observations of Alldredge *et al.* (1990) and Al Ani *et al.* (1991). These features are the result of the form of the product $R(d)p(d, x)m(d, x)$ which, as displayed in Figure 1, is high for large aggregates with maxima that are not close to aggregate surface. Consequently, if a population of aggregates disintegrates in its basic particles, it will not do so in a single step, but in a cascade fashion.

We examined the consequences of these features in the dynamics of particulate matter by including a break-up term in a more general model (the model is described in Ruiz, 1997). The particle model has the same arrangement of size class as that described above. It incorporates particle aggregation, sedimentation, phytoplankton growth and the break-up term described in this paper. All the parameters of the model were held fixed in the standard conditions described in Table 1. With these standard conditions,

Table 1

*Standard value of different parameters of the model.*

| | |
|---|---|
| Mixed layer depth | 50 m |
| Initial dissolved nitrogen | 8 µM |
| $\varepsilon$ (Brainerd and Gregg, 1993) | $3 \times 10^{-5}$ $cm^2\ s^{-3}$ |
| Diameter of the basic particles | 20 µm |
| Size range studied | 20 µm to 5.5 cm. |
| Number of size classes | 18 |
| Stickiness | 0.2 |
| C (Hill, 1992) | 100 |
| Local truncation error | < 0.001 µM |
| $K/\sigma$ | 0.1 $(cm^{-1}\ s^{-1})/(dyn\ cm^{-2})$ |

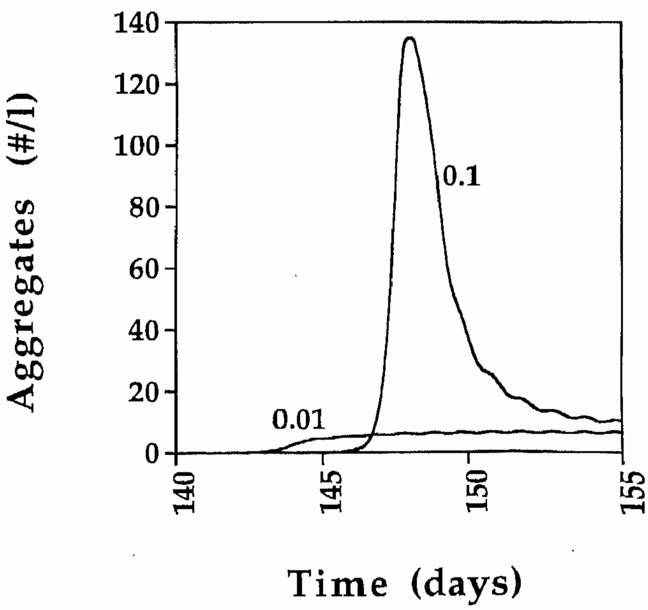


Figure 3

*Sensitivity analysis to determine a proper value for the factor* $K/\sigma$*. The Figure shows the evolution in the concentration of aggregates larger than 500 µm during a phytoplankton bloom for different values of the* $K/\sigma$ *factor. Numbers marking the lines are the different values of the factor* $K/\sigma$*. Aggregate concentrations for* $K/\sigma$ *equal to zero, 0.001 and 1, although plotted, are so low that they cannot be observed in the figure. Other parameters in the particle model were held constant at the standard values of Table 1. The x-axis shows the time of the year (assuming a year of 365 days and assigning the number 1 to the first day of January).*

the model generates a bloom of phytoplankton where cells aggregate according to the kernels and efficiencies described by Hill (1992). We performed a sensitivity analysis by changing the value of $K/\sigma$, the only unknown factor in the break-up model. We also checked for the results of the particulate model when $K/\sigma$ is equal to zero, *i.e.* no break-up of aggregates is included in the model. Figure 3 shows that when $K/\sigma$ is equal to zero the concentrations of aggregates predicted by the model are very low. This is a striking result, as one would expect that, without a break-up term, the formation of aggregates would be favoured rather than diminished. This feature is due to the fact that the model without break-up rapidly eliminates the aggregates as too-large particles, *i.e.* particles bigger than the aggregates usually observed in surface waters and which, consequently, are not included in the model. A similar situation occurs when the value of $K/\sigma$ is too low, for example when $K/\sigma$ is equal to 0.001 or smaller. On the other hand, if the factor $K/\sigma$ is too high ($K/\sigma = 1$), then the concentration of aggregates predicted by the model is also very low, since the strong break-up term does not allow the formation of aggregates. A value of 0.1 ($cm^{-1}$ $s^{-1}$)/(dyn $cm^{-2}$) for $K/\sigma$ allows the formation of aggregates at similar concentrations to those observed in the upper ocean (Lampitt *et al.*, 1993). This value should not be regarded as absolute in the sense that it depends on the model in which it is implemented. Therefore, it could be either too low or too high for a different aggregation model, since it has been found through calibration. However, once the value of $K/\sigma$ is fixed in a model, it is independent of environmental conditions. Thus, for studying particle dynamics, this factor is constant once a proper value for the model is chosen.

After the standard value for $K/\sigma$ was fixed, we tested the effect that an increase in the turbulence level has on the dynamics of aggregates. For this experiment, we generated a bloom of aggregates and then increased the value of $\varepsilon$ from $3 \times 10^{-5}$ to $5 \times 10^{-2}$ $cm^2$ $s^{-3}$ during one day. Figure 4 displays the results of the experiment: the concentration of aggregates sharply decreases during the turbulent period whereas in the model without the turbulent event, there is a clear peak in the concentration of aggregates and a further decrease due to sedimentation.

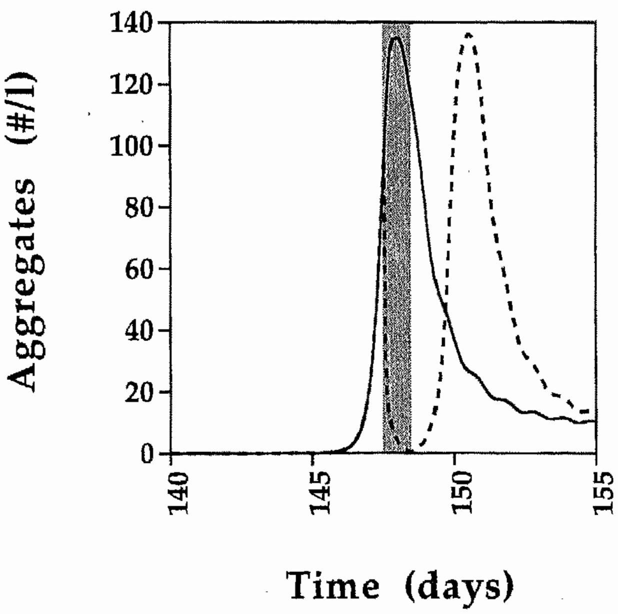


Figure 4

*Response to an increase in turbulence levels of the particle model with a break-up term. The figure shows the evolution in the concentration of aggregates larger than 500 µm during a turbulence event. The value of $\varepsilon$ was $3 \times 10^{-5}$ $cm^2$ $s^{-3}$ throughout the experiment, except for the turbulent period (day 147), when this value was $5 \times 10^{-2}$ $cm^2$ $s^{-3}$ (broken line only). The value of $\varepsilon$ in the turbulent event is of the order of the values reported by Riebesell (1992) for the rupture of aggregates in surface waters ($\sim 10^{-3}$ $cm^2$ $s^{-3}$). The shaded zone delimits the turbulent period.*

## DISCUSSION

Since McCave (1984) published the governing equation for particle dynamics in the pelagic ecosystem, the effort to implement these equations in models has been focused on the aggregation process, whereas the break-up of marine particles has either been neglected or modelled through very simple equations. This neglect has been due partly to confidence that particulate models without a break-up term do not predict the formation of too-large particles. This confidence relied on the high sinking velocity of large particles, the implication being that they would be lost from the water column before the formation of too-large particles.

Our results show that models without a break-up term can fail to predict the formation of marine aggregates at the concentrations observed during blooms. This results from the very short timescale involved in the aggregation of large particles (Riebesell, 1992), which implies the formation of particles which are unrealistically large and are not included in the model. This feature also implies that some models without a break-up term could not conserve the mass if they constantly lose too-large particles during strong aggregation events.

Another reason why break-up has not been included in particle models is the lack of a set of equations reflecting the geometry of the breaking process as aggregation kernels do for aggregation. Attempts to create this set of equations for suspended particles exist in the engineering literature of waste water treatment. Most of them determine the maximum size of stable aggregates rather than the redistribution of mass along the size spectrum resulting from the break-up of aggregates (Tomi and Bagster, 1978; Tambo and Hozumi, 1979; François, 1987). In other studies, the problem of finding the size spectrum that results from the break-up of aggregates, is analysed under a probabilistic assumption (Pandya and Spielman, 1982). Thus, in the fracture of aggregates, it is assumed that, because there are many factors interacting when a floc is broken (*i.e.* turbulence forces, inter-particle forces, floc morphology), an appeal to the central limit theorem leads to the Gaussian distribution for the sizes of the particles resulting after the break-up.

The model we have proposed in this paper constitutes an attempt to reproduce the break-up process of aggregates according to a probabilistic approach that uses dimensional and simple geometric considerations. This formulation does not considers neither the distribution of forces within the aggregate (Sonntag and Russel, 1987), nor their rheological properties (Jenkinson *et al.*, 1991). Nevertheless, the manner in which the model is formulated allows for the inclusion of further processes in the functions $R(d)$ and $p(d, x)$, as well as for their substitution by other functions based on empirical observations. Despite the simplicity of the model, it is able to incorporate three factors that control the rate of rupture of aggregates of different sizes. First, turbulence exerts stronger shear forces on larger aggregates. This is a result of their greater size which widens the surface on which the stress of the fluid flow is exerted. Second, large aggregates are weaker than small ones because their porosity increases and therefore the proportion of constituent matter keeping the aggregate bound decreases. Third, the rate of rupture of an aggregate is considered to be dependent on the position where the rupture takes place, since the sectional area keeping the aggregate bound decreases as the surface of the aggregate is approached. Also, the shear force acting on the surface of the aggregates decreases as we approach the pole of the aggregate. This third factor allows us to determine the size distribution resulting from the break-up of aggregates. Therefore, the model does not distinguish explicitly between fracture and erosion of aggregates, since the same function, $R(d)\,p(d, x)\,m(d, x)$, serves to predict the formation of particles of any size after aggregate break-up.

The equations resulting from the formulation of these factors (eqs. (9) and (10)) are rather complicated for implementation in a model of particles. However, the coefficients of eq. (11) are the elements of an upper triangular matrix. These elements do not depend on the concentration of particles. Consequently, in order to include the break-up process in a more general model, they need to be calculated only once. Moreover, the matrix hey form represents a system of equations that can be inalytically solved for implementation in particle models. This is recommendable, since the timescales involved in the rupture of large aggregates are very short when compared to those involved in other processes of the ecosystem (such as growth, sedimentation or grazing). Consequently, the integration of the general model becomes inefficient (even when stiff methods for solving differential equations are used) if the system of equations (11) is not analytically integrated at each time step.

The stiffness in the equations also arises from the rapid decay rate of large particles when compared to small ones. This difference in the decay rate is consistent with the empirical results obtained by Alldredge *et al.* (1990) and Riebesell (1992) for the break-up of marine aggregates. Alldredge *et al.* concluded through laboratory experiments that break-up is not an important process controlling the dynamics of aggregates in sea water, while Riebesell, on the basis of field observations, concluded the opposite. According to Riebesell, this apparent contradiction is explained by the different size of the aggregates studied in each case. Our results support this conclusion (*i.e.* there is no contradiction between Alldredge *et al.* and Riebesell results) as a consequence of the great differences in break-up rates predicted by the model for large and small aggregates (Fig. 2*d*). These differences in break-up rates could also explain the fact that most of the marine snow (aggregates larger than 0.5 mm) is composed of aggregates smaller than 1 mm and that the size of aggregates increases at density discontinuities where turbulence is usually damped MacIntyre *et al.* (1995).

The formulation presented for the turbulence dependence of rupture rates can also be useful to analyse other experimental observations, such as the break-up model derived by Jackson (1995) which is based on empirically-fitted parameters. Jackson considered that the break-up rate of a certain size class in his model equals $c_3 \times c_4^i$, where $c_3$ and $c_4$ are empirically fitted parameters (their values are 13 and 1.31 respectively) and where $i$ is the class number. For dimensional consistency among different size classes, $c_4$ must be dimensionless. It is the ratio of break-up rates between consecutive size classes. The ratios calculated with expressions in eq. (12) range from 1.54 to 2.46 and have an average value of 1.88 for our arrangement of size classes. This average value is not too different from the empirical value of 1.45 found by Jackson for $c_4$. Moreover, as $c_4$ is dimensionless, $c_3$ must have dimensions of $T^{-1}$ which makes it possible that its value, as in our formulation, is a linear function of turbulent shear, $(\varepsilon/v)^{1/2}$

The high dependence found in the concentrations of aggregates on turbulence levels, both in the modelling results presented in this paper and in field observations (Riebesell, 1992), can be explained by the manner in which break-up depends on $\varepsilon$. As the solution of the system of equations (11) is of the type $e^A$ ($A$ being the upper triangular matrix referred to above), the dependence of marine snow concentration on turbulence levels is also of the type $e^{(\varepsilon^{1/2})}$ ($\varepsilon^{1/2}$ multiplies each of the elements of the matrix $A$). Consequently, an increase in turbulence levels of the mixed layer produces an exponential decrease in the concentration of marine snow. This explains the rapid change in marine snow concentrations following changes in turbulence levels. Aggregation by shear also depends on the turbulence levels, as turbulence determines the shear at small scales. Therefore, aggregation also increases with increasing turbulence and one might expect that increasing aggregation would cancel out the additional break-up during a turbulence event. To analyse this point, we must remember that the kernel for shear aggregation scales with $\varepsilon^{1/3}$ for aggregates of sizes similar or larger than the Kolmogorov length (Hill *et al.*, 1992). On the other hand, the break-up rate for these sizes, in which rupture is an important process, scales with $\varepsilon^{1/2}$. Therefore, in the case of increasing turbulence levels, the rise in the break-up rate induced by turbulent shear on large aggregates will be higher than the rise in the aggregation rate generated by turbulent shear.

These features demonstrate the need to model the break-up of aggregates in order to understand the dynamics of marine particles. The model we have proposed in the present paper constitutes an attempt to broach this question. We provide a set of equations designed to complete the general dynamic equation presented by McCave (1984) and to close the governing equation for the dynamics of pelagic particles. Although the model is based on very simple geometrical and dimensional considerations, its output is consistent with the available empirical results on aggregate break-up. Also, to some extent, the model provides a theoretical explanation for those results.

## Acknowledgements

This work was supported by an M.E.C./Fleming grant from the Ministerio de Educacion y Ciencia and by CICYT proyect n° AMB93-0614-C02-01. We wish to thank Dr. M.J.R. Fasham, Dr. P. S. Hill, Dr. R.S. Lampitt, Dr. I.J. Totterdell and colleagues at the James Rennell Centre for Ocean Circulation for their helpful comments during the development of this manuscript. The suggestions of anonymous referees greatly improved the final version.

## REFERENCES

**Adler P.M., P.M. Mills** (1979). Motion and rupture of a porous sphere in a linear flow field. *J. Rheol.* **23**, 25-27.

**Al Ani S., K.R. Dyer, D.A. Huntley** (1991). Measurement of the influence of salinity on floc density and strength. *Geo-Marine Letts.* **11**, 154-158.

**Alldredge A.L., C.C. Gotschalk** (1988). In situ settling behaviour of marine snow. *Limnol. Oceanogr.* **33**, 339-351.

**Alldredge A.L., C.C. Gotschalk** (1989). Direct observations of the mass flocculation of diatom blooms : characteristics, settling velocities and formation of diatom aggregates. *Deep-Sea Res.* **36**, 159-171.

**Allredge A.L., T.C. Granata, C.C. Gotschalk, T.D. Dickey** (1990). The physical strength of marine snow and its implications for particle disaggregation in the ocean. *Limnol. Oceanogr.* **35**, 1415-1428.

**Bader H.** (1970). The hyperbolic distribution of particle sizes. *J. Geophys. Res.* **75**, 2822-2830.

**Brainerd K.E., M.C. Gregg** (1993). Diurnal restratification and turbulence in the oceanic surface mixed layer. 1. Observations. *J. Geophys. Res.* **98**, 22645-22656.

**Delichatsios M.A., R.F. Probstein** (1975). Coagulation in turbulent flow: Theory and experiment. *J. Colloid Interface Sci.* **51**, 394-405.

**François R.J.** (1987). Strength of aluminium hydroxide flocs. *Wat. Res.* **21**, 1023-1030.

**Hill P.S.** (1992). Reconciling aggregation theory with observed vertical fluxes following phytoplankton blooms. *J. Geophys. Res.* **97**, 2295-2308.

**Hill P.S., A.R.M. Nowell, P.A. Jumars** (1992). Encounter rate by turbulent shear of particles similar in diameter to the Kolmogorov scale. *J. Mar. Res.* **50**, 643-668.

**Hunt J.R.** (1986). Particle aggregate breakup by fluid shear. In: Estuarine cohesive sediment dynamics. *Lecture Notes on Coastal and Estuarine Studies*, **6**, A.J. Mehta, ed., 85-125.

**Jackson G.A.** (1990). A model of the formation of marine algal flocs by physical coagulation processes. *Deep-Sea Res.* **37**, 1197-1211.

**Jackson G.A., S.E. Lochmann** (1992). Effects of coagulation on nutrient and light limitation of an algal spring bloom. *Limnol. Oceanogr.* **37**, 77-89.

**Jackson G.A.** (1995). Comparing observed changes in particle size spectra with those predicted using coagulation theory. *Deep-Sea Res.* **42**, 159-184.

**Jenkinson I.A., B.A. Biddanda, C.M. Turley, P.C. Abreu, U. Riebesell, V.S. Smetacek** (1991). Rheological properties of marine organic aggregates: importance for vertical flux, turbulence and microzones. *Oceanologica Acta*, vol. sp. **11**, 101-107.

**Kranck K., T. Milligan** (1980). Macroflocs: production of marine snow in the laboratory. *Mar. Ecol. Progr. Ser.* **3**, 19-24.

**Kranck K., T. Milligan** (1988). Macroflocs from diatoms: in situ photography of particles in Bedford Basin, Nova Scotia. *Mar. Ecol.Progr. Ser.* **44**, 183-189.

**Lampitt R.S., W.R. Hillier, P.G. Challenor** (1993). Seasonal and diel variation in the open ocean concentration of marine snow aggregates. *Nature* **362**, 737-739.

**MacIntyre, A.L. Alldredge, C.C. Gotshalk** (1995). Accumulation of marine snow at density discontinuities in the water column. *Limnol. Oceanogr.* **40**, 449-468.

**McCave I.N.** (1984). Size spectra and aggregation of suspended particles in the deep ocean. *Deep-Sea Res.* **31**, 329-352.

**Pandya J.D., L.A. Spielman** (1982). Floc breakage in agitated suspensions: theory and data processing strategy. *J. Coll. Interface Sci.* **90**, 517-531.

**Riebesell U.** (1992). The formation of large marine snow and its sustained residence in surface waters. *Limnol. Oceanogr.* **37**, 63-76.

**Riebesell U., D.A. Wolf-Gladrow** (1992). The relationship between physical aggregation of phytoplankton and particle flux: a numerical model. *Deep-Sea Res.* **39**, 1085-1102.

**Ruiz J.** (1997). What generates daily cycles of marine snow? *Deep-Sea Res.* In press.

**Shanks A.L., J.D. Trent** (1980). Marine snow: sinking rates and potential role in vertical flux. *Deep-Sea Res.* **27**, 137-143.

**Sonntag R.C., W.B. Russel** (1987). Structure and break-up of flocs subjected to fluid stresses. *J. Coll. Interface Sci.* **115**, 379-389.

**Tambo N., Y. Watanabe** (1979). Physical characteristics of flocs-I. The floc density function and aluminium floc. *Wat. Res.* **13**, 409-419.

**Tambo N., H. Hozumi** (1979). Physical characteristics of flocs-II.Strength of floc. *Wat. Res.* **13**, 421-427.

**Tomi D.T., D.F. Bagster** (1978). The behaviour of aggregates in stirred vessels. Part I-Theoretical considerations on the effects of agitation. *Trans. Inst. Chem. Engin.* **56**, 1-18.