Meteorological tides and their relationship with tidal semidiurnal residues*

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SUMMARY: Short period modulations of less than one month are studied for M_2 tidal waves in water level records taken from the Gran Canaria Island Shelf off the island's east coast). From the relation ship among modulations with low frequency oscillation observed in this zone (Bruno, 1993), a mechanism based on non-linear interactions of meteorological low frequency oscillation and M_2 signal is proposed. This interaction explains a large amount of residual semidiurnal variance in the water level data.

Key words: Hydrodynamic, tides, atmosphere-ocean interaction, shallow water, non linear phenomena, North Atlantic.

RESUMEN: LA MAREA METEOROLÓGICA Y SU RELACIÓN CON EL RESIDUO SEMIDIURNO DE MAREA ASTRONÓMICA.– En este trabajo se investiga el origen de las modulaciones de corto período (menos de un mes) que presenta la señal de la onda M_2 en registros de niveles del mar tomados en la plataforma insular de la costa Este de la Isla de Gran Canaria. En virtud de la relación que presentan en esta zona estas modulaciones con la oscilación de baja frecuencia (ω<0.003 c/h) (Bruno, 1993), se propone como mecanismo a través del cual se producen éstas, la interacción no lineal entre la oscilación de baja frecuencia de origen meteorológico y la asociada a la onda M_2. Se concluye que esta interacción no lineal puede explicar gran parte de la varianza del residuo semidiurno que presentan los registros del nivel del mar en esta zona.

Palabras clave: Hidrodinámica, marea, interacción atmósfera océano, aguas someras, fenómenos no lineales, Atlántico Norte.

INTRODUCTION

Non-linear interaction between meteorological and astronomical tides has been investigated for many years by different authors: Proudman (1953), Rossiter and Lennon (1968), Munk *et al.* (1964), Munk and Cartwright (1966), Pugh and Vassie (1976), Prandle and Wolf (1978), Amin (1985), Parker (1991), García Lafuente (1986).

The interaction of a low frequency meteorological tide with an astronomical tide produces additional contributions in tidal frequency bands. Since meteorological forcing frequencies are not constant the interaction may cause a spreading of energy around theoretical spectral lines, *Tidal Cusp*. This mechanism is one of the causes that lead to non-perfect line spectra in water level and current records.

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Our interest is centred on the M_2 distortion analysis found in water level records taken from the Gran Canaria island shelf (off the island's east coast). Evolution of monthly estimations shows M_2 wave modulations of a non astronomical origin with periods shorter than one year (Bruno, 1993).

Some experimental evidence of non-linear interaction between M_2 and low frequency signals are shown by application of complex demodulation on the main semidiurnal tidal wave M_2 frequency to study modulation associated periods of less than a month. Several theoretical considerations allow us to understand semidiurnal residue as the second order solution related to this non-linear interaction. On the other hand the observed semidiurnal residue is expressed as a function of the amplitude and phase lag distortions of M_2 signal in order to be compared with the theoretical second order solution.

Second order quasi-analitical solution for non linear interaction between meteorological low frequency and M_2 waves

The solutions of the equations for shallow water tides, in non-linear terms, can be expressed as a sum of a linear contribution and other terms of higher order. The solution can then be expressed (Amin, 1985) as:

$$\zeta_t = \zeta_1 + \zeta_2 + \textit{higher order terms}$$
$$u_t = u_1 + u_2 + \textit{higher order terms}$$

for the sea level and current velocity respectively. The sub-indices stand for the first and second order solution. Considering the first order solution (ζ_1, u_1) as tidal (i.e. M_2) plus a meteorological low frequency wave, and neglecting the third and higher order terms, the solution can be expressed as:

$$\zeta_t = \zeta_{M_2} + \zeta_l + \zeta_2$$
$$u_t = u_{M_2} + u_l + u_2$$

where sub-indices "M_2" and "l" stand for the first order solution for the M_2 and low frequency wave respectively.

The contributions of the second order solution will be at the following frequencies:
zero; $(\omega_{M2}+\omega_l)$ due to non-linear interaction of M_2 with low frequency band and close to M_2 frequency; $2\,\omega_{M2}$ due to non-linear interaction of M_2 with itself; $2\,\omega_l$ due to non linear interaction of low frequency with itself.

The signal at $(\omega_{M2}+\omega_l)$ implies an additional contribution to the semidiurnal tidal band with some consequences on semidiurnal residue. Residue is computed for water level due to M_2 tidal wave as:

$$\zeta_r = \zeta - \zeta_{M_2}$$

where ζ_r is the residue, $\zeta = \zeta_{M_2} + \zeta_2$ is the M_2 distorted signal and ζ_{M_2} is M_2 wave signal.

Thus, the term of the second order solution associated to the interaction between ζ_{M_2} and ζ_l can be considered as the semidiurnal residue and becomes:

$$\zeta_2 = \zeta_r$$

For a semi-infinite spatial domain of uniform depth, h, bounded by a straight coastline and assuming a barotropic, inviscid and unidirectional flux along the coast not depending on the transversal coast coordinate and neglecting the Coriolis term, the hydrodynamic equations for the second order solution are:

$$\frac{\partial u_2}{\partial t} + u_l \frac{\partial u_{M2}}{\partial x} + u_{M2} \frac{\partial u_l}{\partial x} = -g \frac{\partial \zeta_2}{\partial x} \quad (1)$$

$$h \frac{\partial u_2}{\partial x} + \frac{\partial}{\partial x} [(\zeta_{M2} + \zeta_l)(u_l + u_{M2})] = -b \frac{\partial \zeta_2}{\partial t} \quad (2)$$

Eq. (1) refers to the momentum balance and eq.(2) refers to the mass conservation. The non linear part arising from bottom friction has been disregarded in eq. (1) as only the second order contribution is of interest (Godin, 1994).

After operating with eqs.(1) and (2) the following differential equations for ζ_2 solution are obtained:

$$\frac{1}{gh}\frac{\partial^2 \zeta_2}{\partial t^2} - \frac{\partial^2 \zeta_2}{\partial x^2} = \frac{1}{g}\frac{\partial}{\partial x}\left[u_{M2}\frac{\partial u_l}{\partial x} + u_l \frac{\partial u_{M2}}{\partial x}\right]$$
$$-\frac{1}{gh}\frac{\partial}{\partial t}\left[\zeta_{M2}\frac{\partial u_l}{\partial x} + \zeta_l \frac{\partial u_{M2}}{\partial x} + u_{M2}\frac{\partial \zeta_l}{\partial x} + u_l \frac{\partial z_{M2}}{\partial x}\right] \quad (3)$$

being:

$$\zeta_{M2} = A_{M2} cos(\omega_{M2}\, t - \delta_{\zeta M2}) \quad (4)$$

$$u_{M2} = U_{M2} cos(\omega_{M2}\, t - d_{uM2}) \quad (5)$$

$$\zeta_l = A_l cos(\omega_l\, t - \delta_{\zeta l}) \quad (6)$$

$$u_l = U_l cos(\omega_l\, t - \delta_{ul}) \quad (7)$$

the first order solutions for water level and current velocity of each wave whose amplitudes A_{M2}, U_{M2}, A_l, U_l and phase lags $\delta_{\zeta M2}$, δ_{uM2}, $\delta_{\zeta l}$, δ_{ul} are linear functions of the distance from origin, x.

The right hand of eq. (1) can be written into a more convenient form for further uses as:

$$\frac{1}{gh}\frac{\partial^2 \zeta_2}{\partial t^2} - \frac{\partial^2 \zeta_2}{\partial x^2} =$$

$$A_c cos(\omega_{2+} t - \delta_{1+}) + B_c cos(\omega_{2-} t - \delta_{1-})$$
$$+ C_c sin(\omega_{2+} t - \delta_{1+}) + D_c sin(\omega_{2-} t - \delta_{1-})$$
$$+ E_c cos(\omega_{2+} t - \delta_{2+}) + F_c cos(\omega_{2-} t - \delta_{2-})$$
$$+ G_c sin(\omega_{2+} t - \delta_{1+}) + H_c sin(\omega_{2-} t - \delta_{2-})$$
$$+ A_m cos(\omega_{2+} t - \delta_{3+}) + B_m cos(\omega_{2-} t - \delta_{3-})$$
$$+ C_m sin(\omega_{2+} t - \delta_{3+}) + D_m sin(\omega_{2-} t - \delta_{3-}) \quad (8)$$

where:

$$\begin{aligned} \omega_{2+} &= \omega_{M2} + \omega_l & \omega_{2-} &= \omega_{M2} - \omega_l \\ \delta_{1+} &= \delta_{\zeta M2} + \delta_{ul} & \delta_{1-} &= \delta_{\zeta M2} + \delta_{ul} \\ \delta_{2+} &= \delta_{uM2} + \delta_{\zeta l} & \delta_{2-} &= \delta_{uM2} + \delta_{\zeta l} \\ \delta_{3+} &= \delta_{uM2} + \delta_{ul} & \delta_{3-} &= \delta_{uM2} + \delta_{ul} \quad (9) \end{aligned}$$

and the coefficents from A_c to H_c and from A_m to D_m are parameters whose sub-indices stand for the non-linear term from which they arise; 'm' for convective term of the momentum equation, and 'c' for mass conservation, their values being calculated from the following expressions:

$$A_c = \frac{-\omega_{2+} A_{M2} U_l}{2gh} \delta'_{1+}$$

$$B_c = \frac{-\omega_{2-} A_{M2} U_l}{2gh} \delta'_{1-}$$

$$C_c = \frac{\omega_{2+}}{2gh} [A_{M2} U'_l + U_l A'_{M2}]$$

$$D_c = \frac{\omega_{2-}}{2gh} [A_{M2} U'_l + U_l A'_{M2}]$$

$$E_c = \frac{-\omega_{2+} A_l U_{M2}}{2gh} \delta'_{3+}$$

$$F_c = \frac{-\omega_{2-} A_l U_{M2}}{2gh} \delta'_{3-}$$

$$G_c = \frac{\omega_{2+}}{2gh} [A_l U'_{M2} + U_{M2} A'_l]$$

$$A_m = \frac{1}{2g} [U'_{M2} U'_l + U_{M2} U_l [-\delta'_{uM2} \delta'_{ul} - (\delta'_{uM2})^2 - (\delta'_{ul})^2]]$$

$$B_m = \frac{1}{2g} [U'_{M2} U'_l + U_{M2} U_l [\delta'_{uM2} \delta'_{ul} - (\delta'_{uM2})^2 - (\delta'_{ul})^2]]$$

$$C_m = \frac{1}{2g} [U_l U'_{M2} (\delta'_{ul} + 2\delta'_{uM2}) + U_{M2} U'_l (\delta'_{uM2} + 2\delta'_{ul})]$$

$$D_m = \frac{1}{2g} [U_l U'_{M2} (-\delta'_{ul} + 2\delta'_{uM2}) + U_{M2} U'_l (\delta'_{uM2} + 2\delta'_{ul})] \quad (10)$$

where the apostrophe refers to the derivative with respect to the x axis.

If we admit that the coefficents above are constants between two sections along the x axis separated by a small enough distance, then the solution of eq. (2) can be approximated between these two sections by:

$$\begin{aligned} \zeta_2 = {} & \frac{A_c}{Z_{1+}} cos(\omega_{2+} t - \delta_{1+}) + \frac{B_c}{Z_{1-}} cos(\omega_{2-} t - \delta_{1-}) \\ & + \frac{C_c}{Z_{1+}} sin(\omega_{2+} t - \delta_{1+}) + \frac{D_c}{Z_{1-}} sin(\omega_{2-} t - \delta_{1-}) \\ & + \frac{E_c}{Z_{2+}} cos(\omega_{2+} t - \delta_{2+}) + \frac{F_c}{Z_{2-}} cos(\omega_{2-} t - \delta_{2-}) \\ & + \frac{G_c}{Z_{2+}} sin(\omega_{2+} t - \delta_{2+}) + \frac{H_c}{Z_{2-}} sin(\omega_{2-} t - \delta_{2-}) \\ & + \frac{A_m}{Z_{3+}} cos(\omega_{2+} t - \delta_{3+}) + \frac{B_m}{Z_{3-}} cos(\omega_{2-} t - \delta_{3-}) \\ & + \frac{C_m}{Z_{3+}} sin(\omega_{2+} t - \delta_{3+}) + \frac{D_M}{Z_{3-}} sin(\omega_{2-} t - \delta_{3-}) \quad (11) \end{aligned}$$

where:

$$\begin{aligned} Z_{1+} &= (\delta'_{1+})^2 - \frac{\omega^2_{2+}}{gh} & Z_{1-} &= (\delta'_{1-})^2 - \frac{\omega^2_{2-}}{gh} \\ Z_{2+} &= (\delta'_{2+})^2 - \frac{\omega^2_{2+}}{gh} & Z_{2-} &= (\delta'_{2-})^2 - \frac{\omega^2_{2-}}{gh} \\ Z_{3+} &= (\delta'_{3+})^2 - \frac{\omega^2_{2+}}{gh} & Z_{3-} &= (\delta'_{3-})^2 - \frac{\omega^2_{2-}}{gh} \end{aligned}$$

Taking into account that δ_{uM2} in the zone under study is approximately equal to $\delta_{\zeta M2}$-90° (see Tables 1 and 2) then

$$\sin\theta_{uM2} \simeq \cos\theta_{\zeta M2}$$
$$\cos\theta_{uM2} \simeq \sin\theta_{\zeta M2}$$

where

$$\begin{aligned} \theta_{\zeta M2} &= \omega_{M2}\, t - \delta_{\zeta M2} & \theta_{\zeta M2} &= \omega_{M2}\, t - \delta_{\zeta M2} \\ \theta_{\zeta l} &= \omega_l\, t - \delta_{\zeta l} & \theta_{ul} &= \omega_l\, t - \delta_{ul} \quad (12) \end{aligned}$$

TABLE 1. – Astronomical tide harmonic constants in sea level at Taliarte and La Luz station. A amplitude ; G Greenwich phase lag.

constituent	Taliarte station A(cm)	G(°)	La Luz station A(cm)	G(°)
O_1	4.68	293.14	4.89	293.45
P_1	1.76	30.70	1.84	31.01
K_1	5.91	39.90	6.18	40.21
$2N_2$	2.10	359.84	2.17	0.44
μ_2	2.81	348.55	2.90	349.15
N_2	14.43	13.96	14.90	14.56
υ_2	2.87	14.49	2.96	15.09
M_2	70.59	26.41	72.87	27.01
L_2	1.63	23.31	1.68	23.91
T_2	2.17	35.55	2.24	36.15
S_2	26.61	47.02	27.47	48.59
K_2	7.33	36.31	8.30	47.62

TABLE 2. – Astronomical tide harmonic constants in the predominant direction of current velocity at Taliarte and La Luz station. A amplitude; G Greenwich phase.

constituent	Taliarte station A(cm s^{-1})	G(°)	La Luz station A(cm s^{-1})	G(°)
O_1	1.19	320.80	1.00	251.70
P_1	0.45	59.50	0.38	349.26
K_1	1.51	68.70	1.26	358.46
N_2	1.84	298.78	1.17	14.56
M_2	9.00	311.78	6.00	276.61
S_2	3.39	334.38	2.15	299.19
K2	1.03	333.48	0.65	298.22

eq. (11) can then be expressed as:

$$\zeta_2=[R_{1u_l}\cos\theta_{u_l}+R_{2u_l}\sin\theta_{u_l}+R_{1\zeta_l}\cos\theta_{u_l}R_{2\zeta_l}\sin\theta_{\zeta_l}]\cos\theta_{\zeta M2}$$
$$+[R_{3u_l}\cos\theta_{u_l}+R_{4u_l}\sin\theta_{u_l}+R_{3\zeta_l}\cos\theta_{u_l}R_{4\zeta_l}\sin\theta_{\zeta_l}]\cos\theta_{\zeta M2} \quad (13)$$

where:

$$R_{1u_l}=[\frac{A_c}{Z_{1+}}+\frac{B_c}{Z_{1-}}-\frac{C_m}{Z_{3+}}-\frac{D_m}{Z_{3-}}] \qquad R_{1\zeta_l}=[-\frac{G_c}{Z_{2+}}-\frac{H_c}{Z_{2-}}]$$
$$R_{2u_l}=[\frac{C_c}{Z_{1+}}-\frac{D_c}{Z_{1-}}+\frac{A_m}{Z_{3+}}-\frac{B_m}{Z_{3-}}] \qquad R_{2\zeta_l}=[-\frac{E_c}{Z_{2+}}-\frac{F_c}{Z_{2-}}]$$
$$R_{3u_l}=[\frac{C_c}{Z_{1+}}+\frac{D_c}{Z_{1-}}+\frac{A_m}{Z_{3+}}+\frac{B_m}{Z_{3-}}] \qquad R_{3\zeta_l}=[-\frac{E_c}{Z_{2+}}+\frac{F_c}{Z_{2-}}]$$
$$R_{4u_l}=[\frac{A_c}{Z_{1+}}+\frac{B_c}{Z_{1-}}-\frac{C_m}{Z_{3+}}-\frac{D_m}{Z_{3-}}] \qquad R_{4\zeta_l}=[-\frac{G_c}{Z_{2+}}-\frac{H_c}{Z_{2-}}] \quad (14)$$

Eq. (13) can still be written into a more compact form finally as:

$$\zeta_2 = C_t\cos\theta_{\zeta M2}+S_t\sin\theta_{\zeta M2} \quad (15)$$

with

$$C_t=X_{cu}cos(\theta_{u_l}+\varphi_{cu})+X_{c\zeta}cos(\theta_{\zeta_l}+\varphi_{c\zeta})$$
$$S_t=X_{su}cos(\theta_{u_l}+\varphi_{su})+X_{s\zeta}cos(\theta_{\zeta_l}+\varphi_{s\zeta})$$
$$X_{cu}=(R^2_{1u_l}+R^2_{2u_l})^{1/2} \qquad X_{c\zeta}=(R^2_{1u_l}+R^2_{2u_l})^{1/2}$$
$$X_{su}=(R^2_{3u_l}+R^2_{4u_l})^{1/2} \qquad X_{s\zeta}=(R^2_{3\zeta_l}+R^2_{4\zeta_l})^{1/2}$$

$$\varphi_{cu} = \arctan[-\frac{R_{2u_l}}{R_{1u_l}}] \qquad \varphi_{c\zeta} = \arctan[-\frac{R_{2\zeta_l}}{R_{1\zeta_l}}]$$
$$\varphi_{su} = \arctan[-\frac{R_{4u_l}}{R_{3u_l}}] \qquad \varphi_{c\zeta} = \arctan[-\frac{R_{4\zeta_l}}{R_{3\zeta_l}}] \quad (16)$$

From eq. (13) the second order solution associated to non linear interaction between ζ_l and ζ_{M2} waves, is expressed as a summation of terms consisting of products among sines and cosines of low frequency oscillation arguments; $\theta_{\zeta l}$, θ_{ul} and sines and cosines of M_2 wave argument $\theta_{\zeta M2}$. Therefore, the energy of this solution is located at ($\omega_{M2}\pm\omega_l$) frequencies contributing to semidiurnal residue.

In the next section complex demodulation will be applied to characterize semidiurnal residue with a formulation in accordance with the obtained solution.

Application of complex demodulation in the study of distortions of M_2 tidal wave.

Complex demodulation yields temporal amplitude and phase variations of a given wave associated signal which is expressed (Garret *et al*, 1989) as:

$$s(t)=A_aA_tcos(\omega_at-\delta_a\phi_t)$$

being ω_a, A_a and δ_a the angular speed, amplitude and phase lag of the non-distortioned wave respectively; and A_t and ϕ_t being temporal distortion factors for amplitude and phase lag.

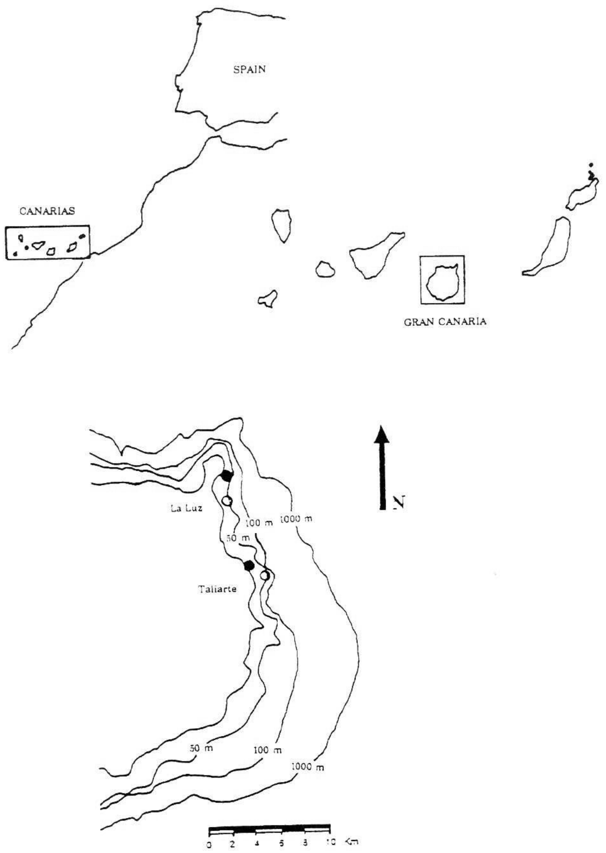

FIG. 1. – Station locations: ● water level recorder, ❍ currentmeter, ❑ meteorological station.

To isolate the distorted signal of M_2 from data records, contributions from N_2, S_2, K_2, μ_2, ν_2 and η_2 must be eliminated. Once this is done, the residual variance can be associated with M_2 tidal wave. Part of the residual variance would be related to non linear interaction between the other semidiurnal waves and low frequency band but can be neglected if we assume that the generating mechanism is non-linear and the distortion amplitude directly proportional to the astronomical wave from which they originate.

The M_2 wave accounts for over 60% of the variance of the semidiurnal tidal signal (Table 1). The rest of the energy is accounted for by the S_2, N_2 and K_2 wave with 22%, 12% and 6% respectively. If the distortion amplitude maintains the same rate as that of the astronomical wave amplitude, the semidiurnal tidal band distortion can be explained in terms of main semidiurnal tidal wave M_2 distortion.

After isolating the M_2 signal from the other constituents, the expression for the distorted M_2 wave is

$$\zeta(t)=A_{M2}A_t cos(\omega_{M2}t-\delta_{\zeta_{M2}}\phi_t)$$

Developing this equation as a function of A_t and ϕ_t by means of Taylor series around the undistorted signal, (A_t=1 and ϕ_t=1) yields

$$\zeta(t)\simeq\zeta(1,1)+\frac{\partial\zeta}{\partial\zeta_t}\Big|_{(1,1)}(A_t-1)+\frac{\partial\zeta}{\partial\zeta_t}\Big|_{(1,1)}(\phi_t-1) \quad (17)$$

Evaluating partial derivatives in A_t=1 and Φ_t=1, M_2 distortion can be characterized as

$$\zeta(t)-\zeta(1,1)\simeq\zeta_{r_a}(t)=A_{M2}\cos(\theta_{\zeta_{M2}})(A_t-1)+\delta_{\zeta_{M2}}A_{M2}\sin(\theta_{\zeta_{M2}})(\phi_t-1) \quad (18)$$

where $\zeta_{r_a}(t)$ can be considered an approximation to $\zeta_r(t)$, the residual signal of M2 wave. M_2 distortion can thus be characterized from Eq. (18).

This approach is satisfactory for the sea level records considered in this work. Figures 2 and 3 show the residual series estimated from $\zeta_r(t)=\zeta_r(t)-\zeta(1,1,t)$ and its approach from Eq. (18) for two different periods yielding an acceptable fitting.

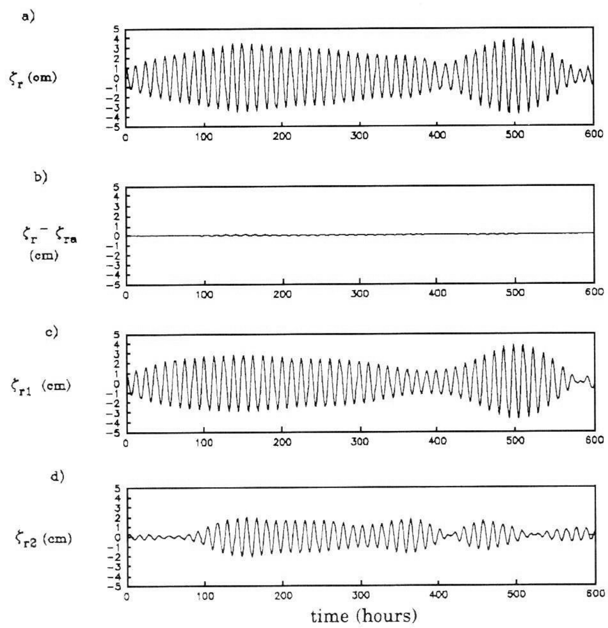

FIG. 2. – La Luz station, period from 6-1-90 to 6-2-90: a) ζ_r: observed semidiurnal residue, b) $\zeta_r-\zeta_{ra}$: difference between observed residue and the approximation based on expression (7), c) ζ_{r1}: amplitude distortion contribution to ζ_r, d) ζ_{r2}: phase distortion contribution to ζ_r.

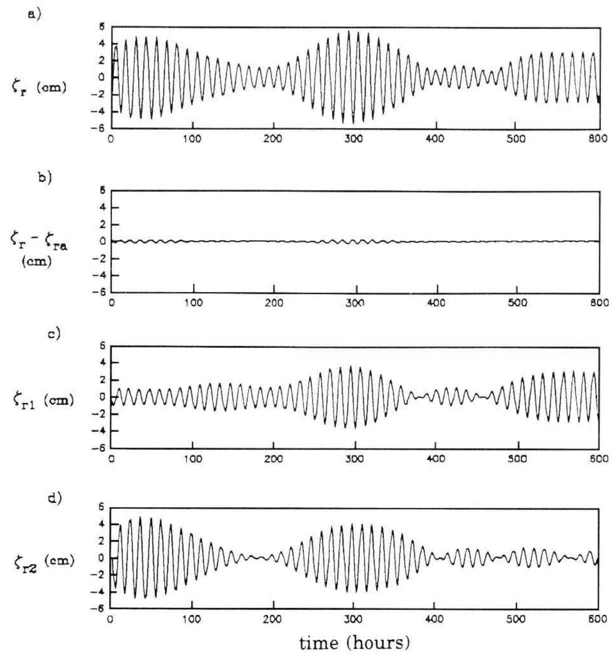

FIG. 3. – La Luz station, period from 30-3-90 to 30-4-90: a) ζ_r: observed semidiurnal residue, b) ζ_r-ζ_{ra}: difference between observed residue and the approximation based in expression (7), c) ζ_{r1}: amplitude distortion contribution to ζ_r, d) ζ_{r2}: phase distortion contribution to ζ_r.

With this procedure the variance of residual can be separated in two parts, one being proportional to the M_2 signal

$$\zeta_{r_1}=C_e\cos\theta_{\zeta_{M2}} \quad (19)$$

and other proportional to a 90° phase lag signal with respect to the first:

$$\zeta_{r_2}=S_e\sin\theta_{\zeta_{M2}} \quad (20)$$

where:

$$C_e=A_{M_2}(A_t-1)$$
$$S_e=\delta_{\zeta_{M2}}A_{M_2}(\phi_t-1) \quad (21)$$

Thus the different contributions to semidiurnal residue from amplitude and phase lag distortions on M_2 wave can be evaluated.

This characterization of the semidiurnal residue through eq. (18) is very useful for investigating the origin of residual signal when this is related with a non linear interaction such as described in section 1. Both formulations, Eq. (15) and Eq. (18), are convergent if both the C_e and S_e series in eqs. (19) and (20) are correlated with the C_t and S_t series in eq. (15). This feature will be considered in the next section.

Computation of amplitude and phase distortions and their relation with low frequency signal.

The application of complex demodulation on the distorted M_2 signal at water level, $\zeta(t)$, yields the series of temporal amplitude and phase lag distortion factors A_t and ϕ_t.

The intervals in which complex demodulation will be applied must be chosen adequately and are those for which low frequency oscillation in sea level and current velocity show a quasi-harmonic behaviour with a frequency ω_l and which, in addition, would be clearly correlated with each other. These conditions will allow an approximation to theoretical series ζ_l and u_l mentioned in the section 1. Intervals were selected from water level and current velocity records taken in La Luz station over a seven month period (Fig. 1).

In accordance with the above conditions the selected period was from 03/30/1990 to 04/30/1990. In this period, which is shown in Fig. 4, both oscillations can be associated, at least for the first two weeks, to an harmonic frequency oscillation of ω_l=2.32 10^{-5} rad s^{-1} corresponding approximately to a periodicity of six days. In addition a clear correlation between both oscillations (ζ_l and u_l) is observed, showing a relative phase lag of close to 180° with each other.

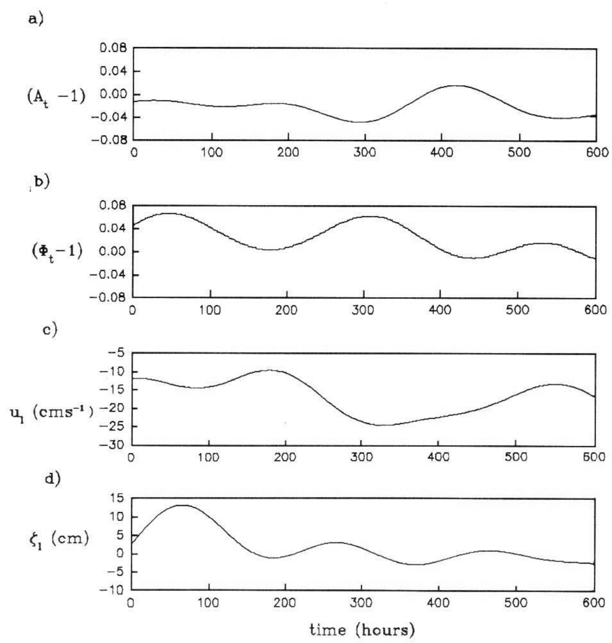

FIG. 4. – La Luz station, period from 6-1-90 to 6-2-90: a) ϕ_t: phase distortion factor, b) A_t: amplitude distortion factor, c) ζ_l: low frequency sea level oscillation refers to mean value over seven months, d) u_l: low frequency current oscillation refers to mean value over seven months.

The constants of the solution of eq. (15) depend on parameters ω_{M2}, A_{M2}, U_{M2}, A'_{M2}, U'_{M2}, δ'_{M2} and δ'_{uM2} associated to M_2 wave and ω_l, A_l, U_l, A'_l, U'_l, $\delta'_{\zeta l}$ and δ'_{ul} associated to the low frequency oscillation.

The parameters associated to the M_2 wave, of angular frequency $\omega_{M2}=1.4\ 10^{-4}$ rad s^{-1}, can be estimated from Tables 1 and 2, taking a distance of 10 Km from Taliarte to La Luz stations, a mean depth h=50 m and a linear variation between these stations for amplitude and phase. Thus the values; $A_{M2}=0.73$ m, $U_{M2}=0.06$ m.s^{-1}, $A'_{M2}= 2.10^{-6}$, $U'_{M2}= 2.10^{-6}$, s^{-1}, $\delta'_{\zeta M2}=10^{-6}$ rad m^{-1} and $\delta'_{uM2}=-6.10^{-5}$ rad m^{-1} can be obtained.

Since we are dealing with parameters associated with low frequency waves, of frequency $\omega_l=2.32\ 10^{-5}$ rad s^{-1}, A_l and U_l can be computed from Fig. 4, taking the average of the differences between maximum and minimum value of the low frequency sea level and current velocity over the first two weeks as an estimation of A_l and U_l, which leads to $A_l=0.025$ m and $U_l=0.075$ m s^{-1}. However, the estimation of A'_l, U'_l and wave numbers $\delta'_{\zeta l}$, δ'_{ul}, can not be performed in such a direct way as no simultaneous records in the stations used were available. For parameters A'_l and U'_l the same values as A'_{M2} and U'_{M2} respectively can be admitted.

An alternative way can be followed to estimate the parameters $\delta'_{\zeta l}$ and δ'_{ul} by taking into account the quasi-static response of low frequency sea level to atmospheric pressure variations in the studied zone. When this quasi-static response exists the relation:

$$\Delta\zeta = \alpha\Delta p_a$$

is practically satisfied, implying that the atmospheric pressure variations around any mean value $\pm\Delta p_a$ mantain a direct proportionality with sea level response $\Delta\zeta_l$, through a negative proportionality coefficent α.

Coefficent α can be estimated for an atmospheric pressure perturbation travelling along a channel with a constant regtangular section and depth, (Proudman, 1953) by:

$$\alpha = \frac{-1}{\rho g(1-\frac{c^2}{gh})}$$

where ρ is sea water density and c is the propagation speed of atmospheric pressure perturbation Δp_a.

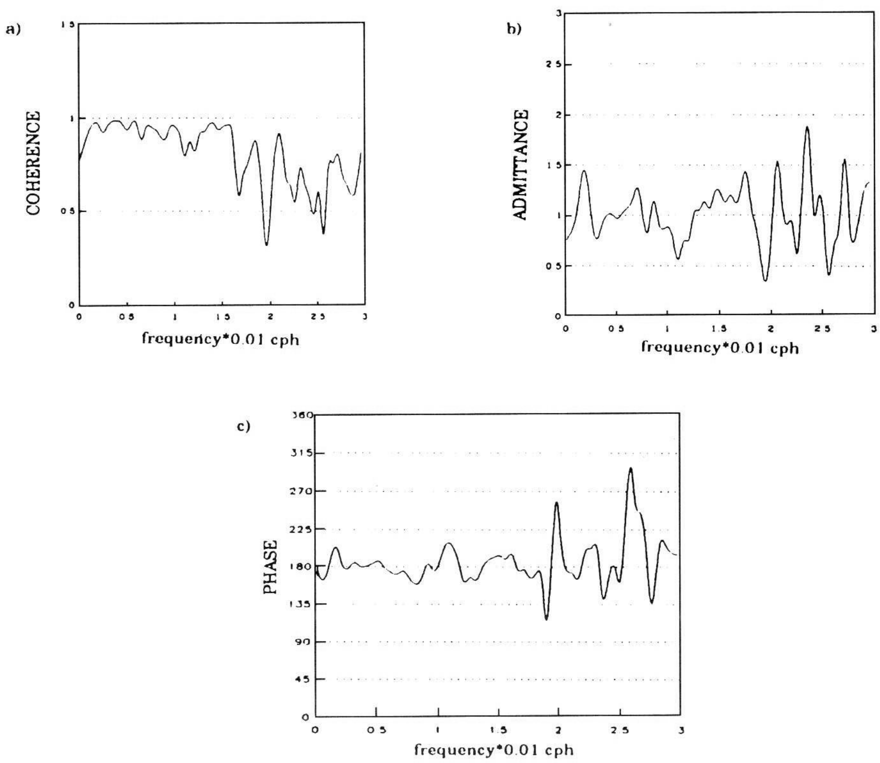

FIG. 5. – Results for cross spectral analysis between atmospheric pressure (mb) and low frequency sea level oscillation (cm) at La Luz station. a) coherence diagram, b) admittance diagram, c) phase diagram.

The modulus of the transfer function, or admittance, from cross spectral analysis between atmospheric pressure (in mb) and low frequency sea level (in cm) series can be read as an experimental estimation of parameter α in oceanic zones where quasi-static response behaviour is found.

The results from cross spectral analysis between atmospheric pressure and low frequency sea level series in La Luz station are shown in Fig. 5. The phase diagram shows that the relative phase lag between both series presents values oscillating ligthly around 180° along the whole of the frequency range. On the other hand, the admittance inside the range which contain the frequency ω_l=6.66 10^{-3} c h^{-1} shows a value close to unity. If this value is used as an estimation of _ and taking the following values; h=50 m, ρ=1025 Kg m^{-3} and g=9.8 m s^{-2}, the propagation speed of the atmospheric pressure perturbation is c_l=1,48 m s^{-1}. Taking now into account that $c_l=\omega_l/k_l$, where k_l is the wave number of the perturbation, we obtain k_l=1.56 10^{-5} rad m^{-1}. Assuming the same wave number for sea level and current velocity response to this perturbation we get $\delta'_{\zeta l}=\delta'_{ul}=k_l$.

The above results lead to the values for constants of eq (15) shown in Table 3. Constants are computed through different values of $\delta'_{\zeta l}$ corresponding to different values of α around unity, first for positive $\delta'_{\zeta l}$ and then for negative ones. Constants $X_{c\zeta}$ and $X_{s\zeta}$ are not presented because they never showed values greater than 10^{-4} m. There is a great sensitivity of the solution to small changes in parameter α However, results for positive $\delta'_{\zeta l}$, lead to S_t amplitude, X_{su}, greater than C_t amplitude, X_{cu}, in all cases. The maximum X_{su} is reached with α=1.02 or $\delta'_{\zeta l}$=6.7 10^{-6} rad m^{-1}. Returning to results of section 2, C_e and S_e or (A_t-1) and (ϕ_t-1) series in eqs. (21) must agree with C_t and S_t behaviour respectively. In fact (A_t-1) amplitude presents a greater value than (ϕ_t-1) amplitude as is showed in Fig. 4.

Since S_t series has almost no delay with respect to low frequency oscillation in current velocity, $\phi_{su}\approx360°$, the phase of (ϕ_t-1) series to low frequency oscillation in current velocity must be close to 360°. In fact, low frequency oscillation in current velocity is approximately in phase with (ϕ_t-1) series, as is shown in Fig. 4, at least, for the two first weeks in the selected period where the assumed harmonic approximation for low frequency oscillation is nearly satisfied. Relative phase of C_t to low frequency oscillation in current velocity is not taken in to account because of the small variation of (A_t-1) in the analyzed period. Since theoretical results of a positive $\delta'_{\zeta l}$ agree with the experimental results, we

TABLE 3. – Constants of analytical solution eq. (15) for different α.

α	$\delta'_{\zeta l}$ 10^{-6} (rad m^{-1})	X_{cu}(cm)	φ_{cu} (deegres)	X_{su}(cm)	φ_{su} (deegres)
1.00	15.60	0.07	-3.95	0.30	-31.35
1.01	8.67	0.13	36.68	1.85	-12.61
1.02	6.70	1.99	70.50	13.90	-9.17
1.03	5.68	0.68	-83.69	8.85	-7.45
1.04	5.05	0.24	-80.25	4.74	-6.30
1.05	4.60	0.18	-81.97	3.67	-5.73
1.10	3.40	0.14	-87.99	2.46	-4.01
1.20	2.50	0.16	-89.71	2.10	-2.87
1.50	1.80	0.18	-88.99	1.94	-2.06
1.50	-1.80	0.67	-86.56	2.20	2.63
1.20	-2.50	1.04	-85.56	2.62	3.67
1.10	-3.40	2.50	-83.69	4.19	5.16
1.05	-4.60	8.93	-82.55	6.86	7.45
1.04	-5.05	4.24	-82.08	2.10	9.74
1.03	-5.68	2.88	-81.44	0.90	20.06
1.02	-6.70	2.69	-80.65	0.98	3.78
1.01	-8.67	8.42	-80.25	9.20	10.32
1.00	-15.60	0.16	-85.56	0.30	31.53

can conclude that, $\delta'_{\zeta l}$ must be positive for the selected period. The relative importance of C_t and S_t amplitudes is changed for negative $\delta'_{\zeta l}$.

These results lead to a dependence of local response of the tide with respect to the meteorological conditions, allowing a semidiurnal residue to be considered as a function of field pressure oscillation. Since the low frequency oscillation behaviour is originated by atmospheric pressure field distribution over an oceanic zone greater than the local domain where data have been taken, the investigated phenomena are dependent on a greater spatial scale than the spatial dimension of the studied zone.

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