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Energetic analysis of tube drawing processes with fixed plug by upper bound method

E.M. Rubio [a,*], C. González [a], M. Marcos [b], M.A. Sebastián [a]

[a] Department of Manufacturing Engineering, National Distance University of Spain (UNED), Juan del Rosal 12, Madrid, Spain

[b] Department of the Mechanical Engineering and Industrial Design, University of Cádiz, c/Chile s/n, Cádiz, Spain

Abstract

In this work, an energetic analysis of thin-walled tube drawing processes in conical converging dies with inner plugs has been made. The method used in the analysis is the upper bound method (UBM). The plastic deformation zone has been modelled by triangular rigid zones (TRZ), with the assumption that the process occurs under plane strain and Coulomb friction conditions.

Keywords: Tube drawing; Fixed inner plug; Upper bound method; Plane strain; Coulomb friction

1. Introduction

Metal tubes are used in a great number of applications, including aerospace, defence, medical, transport and nuclear industries, to name but a few. Tube manufacturing, especially thin-walled ones, usually requires some cold finishing processes in which a tube is drawn through a die so as to bring its diameter, the thickness of its wall or both to standardised values of provision. Sometimes, in order to achieve a more accurate inner diameter dimension, mandrels and plugs are located on the inside [1,2,5–7].

This study, focused on plug drawing (Fig. 1), is based on a series of previous works. First of all, the results obtained by the upper bound method (UBM) were validated with the obtained ones by the slab method (SM), with and without considering friction, the finite element method (FEM) and some experimental results found in the literature [3]. Afterwards, a comparative analysis of the necessary energy to carry out the process for different geometric configurations of the established triangular rigid zones (TRZ) model was made. From the results reached in those analyses, the TRZ pattern that required the least energy to carry out the process was selected for subsequent studies. Additionally, the robustness of that TRZ pattern versus above and below variations of the selected parameter to change the configuration was proven.

2. Upper bound method application

Thin-walled tubes drawing through conical convergent dies with an inner, conical or cylindrical, plug fixed to the draw bench is the process that it is going to be analysed by the upper bound method. Tube inner diameter is considered constant along the process ($D_{\mathrm{Ii}} \approx D_{\mathrm{If}} \approx D$), varying only the thickness from an initial value of h_{i}, to a final one of h_{f}.

The process variables are: the conical convergent die semiangle, α; the fixed conical plug semiangle, β, placed inside of the die, and the tube cross-section area reduction, r. This last one can approached the thickness change of the tube wall, if diameters are sufficiently large, like it has been supposed in this case where $h_{\mathrm{i}} \ll D_{\mathrm{Ii}} \approx D_{\mathrm{If}} \approx D$ and $h_{\mathrm{f}} < h_{\mathrm{i}}$, then $h_{\mathrm{f}} \ll D_{\mathrm{Ii}} \approx D_{\mathrm{If}} \approx D$.

When in a thin-walled tube drawing process with fixed plug there is no appreciable variation of its inner diameter ($h_{\mathrm{i}} \approx h_{\mathrm{f}} \ll D_{\mathrm{Ii}} \approx D_{\mathrm{If}} \approx D$), the material placed between the die and the plug is under plane strain conditions [7]. In such situations, it is possible to use S, the yield stress under plane strain instead of Y, the uniaxial yield stress.

At the die exit, metal is free to undergo transverse or circumferential strains. Then it is under a state of uniaxial stresses rather than under plane strain [5–7]. For that reason, some authors recommend that the plug was slightly larger than that necessary one to obtain the precise dimensions of the tube [1].

The strength that finally limits the last pass is the uniaxial yield stress, Y, and no the yield stress under plane strain, S. Although, the plane strain conditions stay in the real deformation zone. It is supposed the breakage is reached as soon as the

* Corresponding author. Tel.: +34 91 398 82 26; fax: +34 91 398 82 28.
E-mail address: erubio@ind.uned.es (E.M. Rubio).

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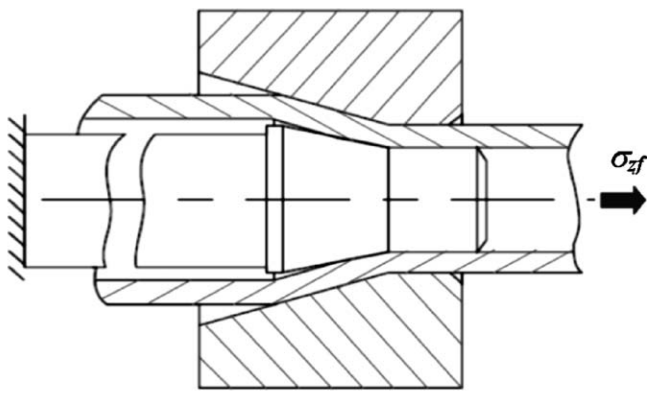

Fig. 1. Plug drawing.

uniaxial yield stress, Y, appears in the tube drawing. The condition expresses the limit of the tubes drawing:

$$\frac{\sigma_{zf}}{S} = \frac{Y}{S} = 0.866 \tag{1}$$

On the other hand, the existing friction between the interfaces: die-tube outer surface and plug-tube inner surface have been considered of Coulomb type. This kind of friction condition considers the shear stress, τ, is proportional to the existing pressure, p, between the surfaces in contact according to the expression $\tau = \mu p$, where the proportionality coefficient, μ, is called Coulomb friction coefficient. In this case, representing the friction value at the interfaces above, the coefficients μ_1 and μ_2, have been used respectively.

The deformation zone has been modelled by three TRZ as it is shown in Fig. 2. According to the literature, the multiple TRZ pattern provides quite low solutions of the UBM [1]. It has been considered that the individual reductions obtained in each one of those zones are identical. Its value can be calculated through the following expression [4]:

$$r_1 = r_2 = r' = 1 - \sqrt{1 - r} \tag{2}$$

With a tube made of rigid-perfectly plastic material, modelling the plastic deformation zone by means of three TRZ (see Fig. 2) and applying the UBM to the drawing process it is possible to write the next expression for calculating the required power to carry out the process:

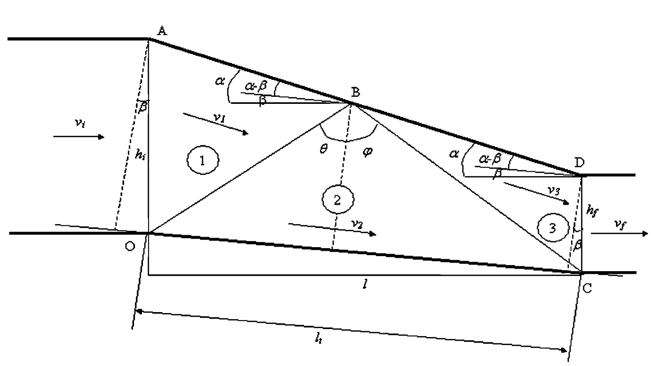

Fig. 2. Plastic deformation zone modelled by three TRZ.

$$\begin{aligned}\dot{W}_T = \sigma_{zf}\pi D h_f v_f = 2\pi D(&k\overline{OA}\Delta v_{i1} + k\overline{OB}\Delta v_{12} + k\overline{BC}\Delta v_{23} \\ &+ k\overline{CD}\Delta v_{3f} + \mu_1 p\overline{AB}v_1 + \mu_2 p\overline{OC}v_2 + \mu_1 p\overline{BD}v_3)\end{aligned} \tag{3}$$

where σ_{zf} is the stress at the die exit; D and h_f, the diameter and the final thickness of the tube respectively; v_f, the velocity at the tube exit; k, the shear yield stress; $k\overline{OA}\Delta v_{i1}$, $k\overline{OB}\Delta v_{12}$, $k\overline{BC}\Delta v_{23}$, $k\overline{CD}\Delta v_{3f}$, the mechanic effects along the $\overline{OA}$, $\overline{OB}$, $\overline{BC}$ and $\overline{CD}$ discontinuities lines; $\mu_1 p\overline{AB}v_1$ and $\mu_1 p\overline{BD}v_3$ the friction effect between the material in the deformation zone and the die along the $\overline{AB}$ and $\overline{BD}$ and $\mu_2 p\overline{OC}v_2$ the friction effect between the material in the deformation zone and the inner plug along the $\overline{OC}$. Pressures in the die and in the plug have been supposed equal and of value p. This can be justified considering, when the radial forces balance is made, that the contribution to the friction of the die is small and that the main stress can be taken as: $\sigma_1 = \sigma_z$ and $\sigma_2 = -p$; being related in a closed pass without diameter variation:

$$\frac{p}{2k} = 1 - \frac{\sigma_z}{2k} \tag{4}$$

that represents the flow condition under plane strain; v_{ij}, the relative speed between the i and j blocks (the three triangular ones and the rectangular at the entrance and at the exit of the tube).

Keeping in mind the value of the pressure given by the expression (4), the symmetry of the problem and the geometric and cinematic relationships that exist between the segments and the relative velocities (Fig. 2), the Eq. (3) can be written by

$$\begin{aligned}\left(\frac{\sigma_{zf}}{2k}\right)_T = &\frac{\overline{OA}\Delta v_{i1} + \overline{OB}\Delta v_{12} + \overline{BC}\Delta v_{23} + \overline{CD}v_{3f}}{h_f v_f + 2[\mu_1\overline{AB}v_1 + \mu_2\overline{OC}v_2 + \mu_1\overline{BD}v_3]} \\ &+ \frac{2[\mu_1\overline{AB}v_1 + \mu_2\overline{OC}v_2 + \mu_1\overline{BD}v_3]}{h_f v_f + 2[\mu_1\overline{AB}v_1 + \mu_2\overline{OC}v_2 + \mu_1\overline{BD}v_3]}\end{aligned} \tag{5}$$

that it represents the adimensional total energy necessary to carry out the process.

3. Energetic analysis

Classic analysis methods such as the homogeneous deformation one or the slab one only provide a low estimate of the necessary energy to carry out a process. The first one only takes into account the necessary energy to carry out the homogeneous deformation and, the second one, that one plus the required one to overcoming the friction. UBM allows evaluating, also, the term due to the internal distortion that suffers the material when it is being deformed. Therefore, the necessary total energy to reduce the thickness of the tube wall can be considered composed by the energy to achieve the homogeneous deformation, the corresponding one to the internal distortion, usually called redundant energy, and the necessary one to overcoming the effect of the friction. Each one of them comes given by the next expressions respectively:

$$\left(\frac{\sigma_{zf}}{2k}\right)_{\mathrm{H}} = \ln\frac{1}{1-r} \tag{6}$$

$$\begin{aligned}\left(\frac{\sigma_{zf}}{2k}\right)_{\mathrm{R}} &= \left(\frac{\sigma_{zf}}{2k}\right)_{\mathrm{T}}\Bigg|_{\mu_1=\mu_2=0} - \left(\frac{\sigma_{zf}}{2k}\right)_{\mathrm{H}} \\ &= \frac{\overline{\mathrm{OA}}\Delta v_{\mathrm{i}1} + \overline{\mathrm{OB}}\Delta v_{12} + \overline{\mathrm{BC}}\Delta v_{23} + \overline{\mathrm{CD}}v_{3\mathrm{f}}}{h_{\mathrm{f}}v_{\mathrm{f}}} \\ &\quad - \ln\frac{1}{1-r}\end{aligned} \tag{7}$$

$$\begin{aligned}\left(\frac{\sigma_{zf}}{2k}\right)_{\mathrm{F}} &= \frac{\overline{\mathrm{OA}}\Delta v_{\mathrm{i}1} + \overline{\mathrm{OB}}\Delta v_{12} + \overline{\mathrm{BC}}\Delta v_{23} + \overline{\mathrm{CD}}v_{3\mathrm{f}}}{h_{\mathrm{f}}v_{\mathrm{f}} + 2[\mu_1\overline{\mathrm{AB}}v_1 + \mu_2\overline{\mathrm{OC}}v_2 + \mu_1\overline{\mathrm{BD}}v_3]} \\ &\quad + \frac{2[\mu_1\overline{\mathrm{AB}}v_1 + \mu_2\overline{\mathrm{OC}}v_2 + \mu_1\overline{\mathrm{BD}}v_3]}{h_{\mathrm{f}}v_{\mathrm{f}} + 2[\mu_1\overline{\mathrm{AB}}v_1 + \mu_2\overline{\mathrm{OC}}v_2 + \mu_1\overline{\mathrm{BD}}v_3]} \\ &\quad - \frac{\overline{\mathrm{OA}}\Delta v_{\mathrm{i}1} + \overline{\mathrm{OB}}\Delta v_{12} + \overline{\mathrm{BC}}\Delta v_{23} + \overline{\mathrm{CD}}v_{3\mathrm{f}}}{h_{\mathrm{f}}v_{\mathrm{f}}}\end{aligned} \tag{8}$$

Table 1
Variables values used in the calculation of the energy

α (°)	β (°)	r	μ_1	μ_2
5	1–4	0.1–0.25	0–0.30	0–0.30
10	1–9	0.1–0.25	0–0.30	0–0.30
15	1–14	0.1–0.20	0–0.30	0–0.30
20	1–19	0.1	0–0.30	0–0.30

4. Applications and results

For the TRZ pattern fixed for $\varphi = 30°$, the expressions (5)–(8) have been calculated for the values collected in Table 1 since, with the calculations carried out to determine the sensibility of the mentioned TRZ pattern, it was possible to see that some variables combinations did not provide possible physically results. For each one of the proven values groups they have been obtained graphics as the collected ones in Fig. 3 for the case of $\alpha = 15°$, $r = 0.10$ and values of μ_1 and μ_2 varying from 0.05 to 0.30.

The obtained results have allowed establishing for each couple (α, r): an interval of β values that make possible the drawing was carried out; the β value that makes minimum the necessary energy to carry out the process, β_{opt}, and the interval of values around that β_{opt} that make the energy used in

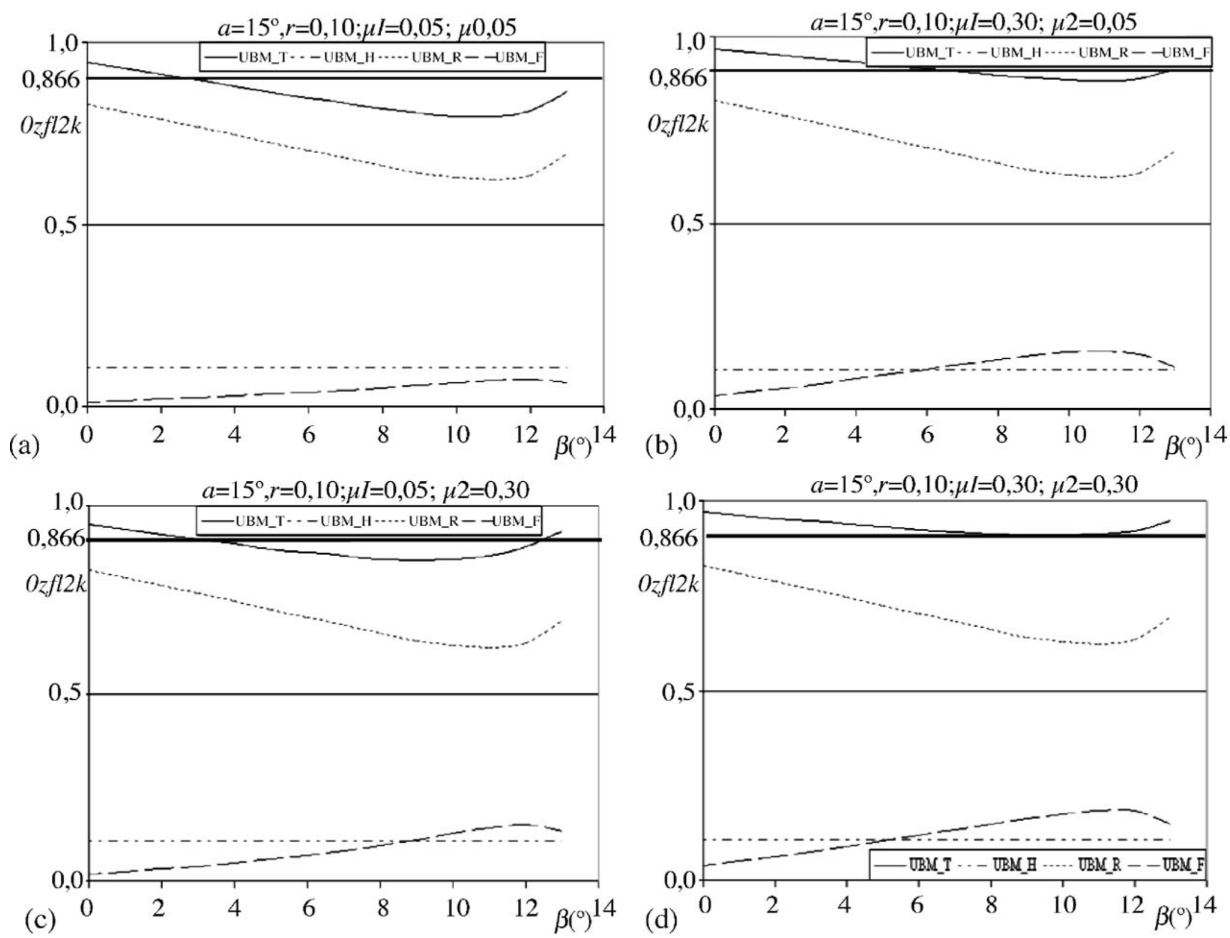

Fig. 3. Total energy (UBM_T), homogeneous one (UBM_H), redundant one (UBM_R) and friction one (UBM_F) vs. β for $\alpha = 15°$, $r = 0.10$. (a) $\mu_1 = \mu_2 = 0.05$; (b) $\mu_1 = 0.05$ and $\mu_2 = 0.30$; (c) $\mu_1 = 0.30$ and $\mu_2 = 0.05$; (d) $\mu_1 = \mu_2 = 0.30$.

Table 2
Intervals of β values that make the process can be carried out, can be carried out in an optimal way and β_{opt}

α (°)	r	β		
		Possible interval (°)	Optimal interval (°)	β_{opt} (°)
5	0.10	0–4	0–2	0
	0.15	0–3	0–1	0
	0.20	0–2	0–1	0
	0.25	0–1	0–1	0
10	0.10	0–9	0–6	4
	0.15	0–8	0–6	2
	0.20	0–6	0–4	0
	0.25	0–3	0–2	0
15	0.10	0–13	6–10	8
	0.15	0–13	5–9	7
	0.20	0–10	0–8	4
20	0.10	10–16	10–16	13

the tubes drawing continues being near to the required minimum one. Such intervals and angles have been collected in Table 2.

5. Conclusions

The energetic analysis of thin-walled tube drawing processes made by the UBM modelling the plastic deformation zone by three TRZ and considering plane strain and Coulomb friction conditions has provided expressions to calculate the adimensional overall energy involved in this kind of process and its components: homogeneous deformation one, redundant work and that required to overcome external friction.

For each group of tested variables, values (α, r, μ_1, μ_2) were been obtained: the interval of values that allows the process to be carried out, the one that enables the process to be carried out in optimal conditions and the value, β_{opt}, that enables the process to be carried out using minimum energy.

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