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Identification of Nonlinear Ship Model Parameters Based on the Turning Circle Test

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This work presents a contribution to solving the problem of identification of ship model parameters using the experimental results from a particular trial test. The innovation of this paper lies in the fact that for this identification purpose it is necessary to know only the turning radius that describes the ship during the performance of the turning test trial. A relatively complex nonlinear model of Norrbin has been chosen as a basis because it represents the ship's dynamics appropriately, as proven through experimental measurements obtained during the course change test. The proposed algorithm of identification of the four ship model parameters is based on an adaptive procedure and the backstepping theory. Another additional coefficient can be determined by an alternative procedure. The knowledge of the true values that characterize the dynamic of a ship is fundamental in the ship steering control that is carried out by autopilots. The simulation results show the suitability of the proposed procedure.

1. Introduction

THE LINEAR THEORY is useful for analyzing the influence of ship features on control-fixed stability as well as on the turning ability of stable ships in the linear range. However, it fails to predict accurately the characteristics of the tight maneuvers that most ships are capable of performing and cannot predict the maneuvers of unstable ships. In a real sense, the presence of different kinds of uncertainty in not precisely known models; the random processes statistics, such as winds, waves, and currents; other exogenous effects on different sailing conditions, such as speed, loading conditions, trim, and so forth; and sailing routes in open sea (deep water) and coastal (shallow) waters with a possible change in the under-keel clearance make necessary the application of techniques that take into account the nonlinear equations that describe the ship's motion, especially in course-keeping and course-changing problems with the presence of unknown parameters which appear linearly in this system of equations.

Unfortunately, there is no completely analytical procedure for predicting the characteristics of these nonlinear maneuvers prior to the conduct of full-scale trials. In the absence of theoretical procedures, an experimental technique using free-running models has been employed for many years. A semitheoretical technique utilizes the experimental test results in conjunction with nonlinear equations of motion expanded to include significant nonlinear terms, as the ship's model employed in this paper. In this sense, a relatively novel identification procedure based on backstepping (Krstić et al. 1995) has been applied with considerable success in axial compressors developed under backstepping designs for throttle and bleed valves (Banaszuk & Krener 1997) and air injection (Behnken & Murray 1997, Protz & Paduano 1997); ship control (Fossen 1994, Ihle et al. 2005, Skjetne & Fossen 2004); route planning (Haro Casado & Velasco 2003); electric machines (induction motor) (Marino et al. 1999); and aircraft control (Härkegard 2001).

There are no definitive international standards for conducting maneuvering trials with ships. Many shipyards have developed their own procedures driven by their experience with consideration of the efforts made by the International Towing Tank Conference (ITTC, Proceedings 1963–1975) and other organizations or institutes (Journée & Pinkster 2001). The Society of Naval Architects and Marine Engineers (SNAME) has produced three guidelines: *Code on Maneuvering and Special Trials and Tests*

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(1950), *Code for Sea Trials* (1973), and *Guide for Sea Trials* (1989). The Norwegian Standard Organization has produced *Testing of New Ships, Norsk Standard* (1985). The Japan Ship Research Association (JSRA) has produced *Sea Trial Code for Giant Ships* (1972) for maneuvering trial procedure and analysis of measurements.

The International Maritime Organization (IMO) has been concerned with the safety implications of ships with poor maneuvering characteristics since the meeting of the subcommittee on Ship Design and Equipment (DE) in 1968. The last resolution adopted by the IMO and by its Maritime Safety Committee (MSF) at its 76th session was the MSC.137(16), which resolves that the adopted provisions supersede the previous annexed to resolution A.751(18) and its circular 644, the title of which is *Explanatory Notes to the Interim Standards for Ship Manoeuvrability*, dated 6 June 1994. The resolution MSC.137(76) has provided circular 1053, *Explanatory Notes to the Standards for Ship Manoeuvrability*, dated 16 December 2002. The Standards are based on the premise that the maneuverability of ships can be adequately judged from the results of typical ship trial maneuvers.

Between the 18 types of maneuvering tests, only the turning test, mainly used to calculate the ship's steady turning radius and to check how well the steering gear performs under course-changing maneuvers; the Z-maneuvering test, used to compare the maneuvering properties and control characteristic of a ship with those of other ships; and the stopping test (crash-stop and low-speed), used to determine the ship's head reach and maneuverability during emergency situations, are recommended by all organizations. In this paper the maneuver of the turning circle test has been chosen as a basis for the identification procedure.

The remainder of this paper is organized as follows: Section 2 deals with nomenclature, referential systems, the modeling problem, and the characterization of the circular movement. Section 3 describes a coarse identification procedure. Section 4 analyzes an identification and control method based on backstepping. Section 5 discusses the simulation results, highlighting some concluding remarks.

2. Ship model

2.1. Coordinate systems

In the process of analyzing the motion of a ship in 2 degrees of freedom (DOF), it is convenient to define two coordinate systems, as indicated in Fig. 1. The moving coordinate frame X_0 Y_0 is conveniently fixed to the ship and is denoted as the body-fixed frame. The origin of this body-fixed frame is usually chosen to coincide with the center of gravity (CG) when CG is in the principal plane of symmetry. The earth-fixed coordinate frame is denoted as X Y. The angle Ψ is the difference between heading and track course, V_L is the forward velocity, V_T is the velocity in starboard direction, and δ is the rudder angle. The coordinates (x, y) denote the ship's position along the track.

2.2. Ship model structure

The elimination of the speed of drift of the ship in the model of Davidson and Schiff (1946) has led to the models of Nomoto et al. (1957). Later, Norrbin (1970) and Bech and Wagner (1969) substituted the linear term of the angular acceleration of the ship for a nonlinear term formed by a third-order polynomial the coefficients of which were determined by Bech's reverse spiral maneuver, in the first- and second-order models of Nomoto, respectively. Based on the kinematic variables defined in Fig. 1, the motion equations that can describe the ship movements are as follows:

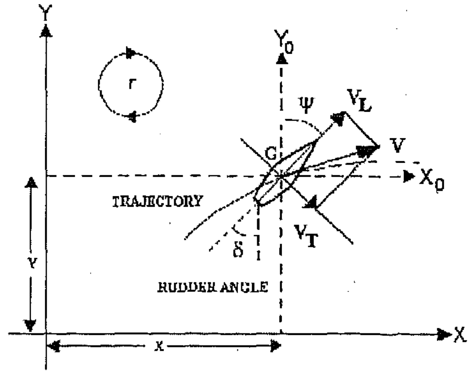

Fig. 1 Variables used for the described ship movement

$$\dot{x} = V_L \cdot \sin\psi + V_T \cdot \cos\psi \quad (1a)$$

$$\dot{y} = V_L \cdot \cos\psi - V_T \cdot \sin\psi \quad (1b)$$

where speed rate in the longitudinal direction or surge speed and the transverse or sway one, V_L and V_T, respectively, are

$$\dot{V}_L = -d \cdot V_L - e \cdot V_\psi^2 + S \quad (2a)$$

$$V_T = -f \cdot V_\psi - g \cdot V_\psi^3 \quad (2b)$$

V_ψ being the ship's speed in the angular sense. Equation (2a) indicates how the propeller's thrust resulting in a forward acceleration S is counteracted by the ship's speed as well as by the turning rate. Using the conventional notation

$$V_\psi = \dot{\psi} = r \quad (3a)$$

the dynamics of the ship can be represented by

$$\dot{r} = -a_0 - a_1 \cdot r - a_2 \cdot r^2 - a_3 \cdot r^3 + c \cdot \delta \quad (3b)$$

where δ is the rudder angle. Equation (3b) represents the well-known nonlinear model describing the steering dynamics proposed by Norrbin, which is an extension of the first-order Nomoto model that has been applied to include nonlinear effects by means of the consideration function that describes the nonlinear nature of ship dynamics and can be expressed in terms of a polynomial expansion as $H_N(r) = -a_0 - a_1 \times r - a_2 \times r^2 - a_3 \times r^3$. In this paper, the turning circle test is used instead of Bech's reverse spiral maneuver, being necessary only for identification purposes to determine the turning radius that is described by the ship during the turning test trial. Nevertheless, the following simplifications are often taken into account:

- Hull symmetry (implies that $a_2 = 0$)
- The dynamic stability is known. This implies that a_1 is known. For a course-stable ship, $a_1 > 0$, whereas for a course-unstable ship, $a_1 < 0$.
- The bias term a_0 is frequently taken as null, being conve-

niently treated as an additional rudder off-set that can be made null by an adequate selection of the integral action in the autopilot design.

In this paper the most general case is considered, that is, $a_0 \neq 0$, $a_2 \neq 0$, and $a_1 > 0$.

The procedure has been applied to a roll-on/roll-off ship (built in the Navantia shipyard in Cádiz, Spain, construction code H81, 2001), the characteristics of which are listed in Table 1 (shown in Fig. 2).

2.3. Description of the circular motion

Note that the transverse speed cannot be considered zero during a circular movement because a nonzero transverse velocity is a prerequisite for turning the ship along a circular path, such as the turning circle, and this condition should be stayed during the entire circular trajectory. However, it seems reasonable to suppose that $V = V_L >> V_T$.

Under this assumption equations (1a) and (1b) are

$$\dot{x} \approx V_L \cdot \sin \psi \tag{4a}$$

$$\dot{y} \approx V_L \cdot \cos \psi \tag{4b}$$

In a radial movement the dynamic equations are (see details in Fig. 3)

$$x = x_0 + R \cdot \sin \gamma \tag{5a}$$

$$y = y_0 + R \cdot \cos \gamma \tag{5b}$$

Resolving equations (4) and (5) in terms of $\dot{R}$ and $\dot{\gamma}$ can be easy to prove that the temporal variations of the ship angular position and the turning radius, after considering that in a radial motion ($\dot{x}_0 = \dot{y}_0 = 0$), are given by

$$\dot{\gamma} = \frac{V_L}{R} \cdot \sin(\psi - \gamma) \tag{6a}$$

$$\dot{R} = V_L \cdot \cos(\psi - \gamma) \tag{6b}$$

which are the fundamental cinematic equations for the description of a ship's radial movement.

3. Statement of the problem under space state description

3.1. Space state equations

For this purpose, the following state variables are introduced: $x_1 = R - R_d = \tilde{R}$, where R_d is the radius that is described by the

Table 1 General description of the ship tested

Length overall	183.3 m
Length between perpendiculars	170.0 m
Deadweight	7,456 metric tons
Displacement	19,994 metric tons
Breadth (molded)	28.7 m
Normal ballast draft (overall)	5.2 m
Propellers	2
Type of rudder	Becker (2 units)
Engine (number of units)	2 per shaft
Engine (power output)	4 × 6,000 kW
Maneuvering speed in ballast condition (maximum ahead)	22.68 knots

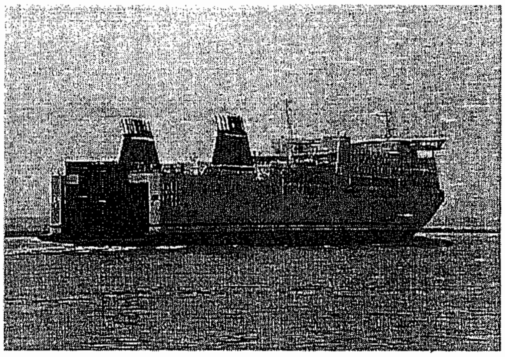

Fig. 2 The ship tested in this study

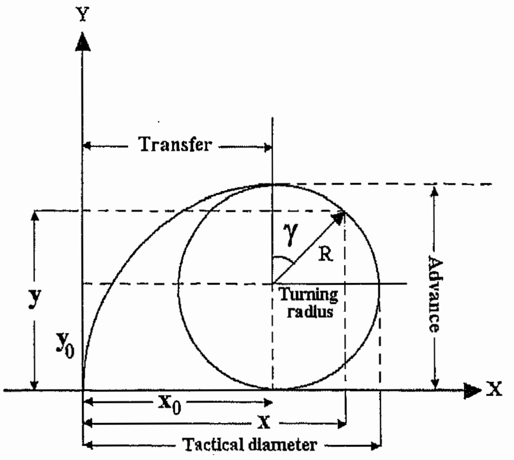

Fig. 3 Turning circle test

ship during the realization of the turning circle test, R is the actual one, and $\tilde{R}$ represents the error between these two variables; $x_2 = \psi$ (yaw angle); $x_3 = r$ (yaw rate); $x_4 = \gamma$ (angular position in the circular motion). With those choices the state equations are as follows:

$$\dot{x}_1 = V \cdot \cos(x_2 - x_4) \tag{7a}$$

$$\dot{x}_2 = x_3 \tag{7b}$$

$$\dot{x}_3 = -\sum_{i=0}^{3} a_i \cdot x_3^i + c \cdot \delta \tag{7c}$$

$$\dot{x}_4 = \frac{V}{x_1 + R_d} \cdot \sin(x_2 - x_4) \tag{7d}$$

which are the basic equations for the description of ship dynamics during the realization of the turning circle test for a constant rudder angle δ.

3.2. Previous model parameters identification

To establish the adequacy of the proposed procedure of identification, it is necessary to know the true values of the parameters (assumed constants) that appear in the dynamic equations. For this purpose, the goal is to reduce the difference between the experimental values obtained from the experimental course change speed (shown in Table 2) and the solutions of the nonlinear dif-

Table 2 Course change test results in normal ballast condition, maximum ahead speed and 10 deg of rudder

t (s)	V_L (m/s)	V_T (m/s)	r (rad/s)
0	11.67	0	0.00143
19.67	10.15	5.76	0.01327
32.00	9.94	6.10	0.01438
44.38	9.74	6.43	0.01408
56.88	9.52	6.74	0.01370
69.67	9.33	7.01	0.01363
82.38	9.15	7.24	0.01373
95.25	9.00	7.43	0.01366
108.00	8.87	7.58	0.01342
121.00	8.75	7.72	0.01377

These experimental values were provided by Navantia (2001) from the sea test trials of a ship of new construction (particulars listed in Table 1), in accordance with the maneuvering specifications of IMO Resolution A.601 (15).

ferential equation (3b) by means of an appropriate selection of the equation's parameters. The procedure is based on the following procedures and algorithms:

- A backward-Euler integration algorithm with a step size of 1 second.
- An optimization algorithm of Powell (Darnell & Margolis 1990) with an optimization criteria ITAE (assuming the ITAE performance criterion as the integral of the square error of the error)
- A trial-and-error procedure.

The resulting parameters are shown in Table 3, and in Fig. 4, the experimental yaw rate is compared with the theoretical ones, showing an excellent agreement between them. In Figs. 5 and 6, the temporal variation of the transverse speed and the longitudinal one are shown. The purpose of this paper is to design a systematic procedure for finding an identification algorithm that does not depend, even partially, on a heuristic method, such as the one used to know the true values of the ship's model parameters, so that it will be possible to confirm the usefulness of the proposed procedure. The procedure described in this paper can be used as a new and alternative estimation method to the traditional ones: continuous least-squares, recursive least-squares, recursive maximum likelihood, and state augmented Kalman filter.

Table 3 Values of parameters

Parameter	Value	Units
a_0	$3.10284 \cdot 10^{-3}$	rad/s^2
a_1	$1.00982 \cdot 10^{-2}$	l/s
a_2	1	l/rad
a_3	0.77	s/rad^2
c	$2.45483 \cdot 10^{-2}$	l/s^2
d	$9.93 \cdot 10^{-3}$	l/s
e	$-4.86 \cdot 10^{-3}$	m/rad^2
S	0.87	m/s^2
f	-98.1	m/rad
g	$-4.23 \cdot 10^{-5}$	ms/rad^3

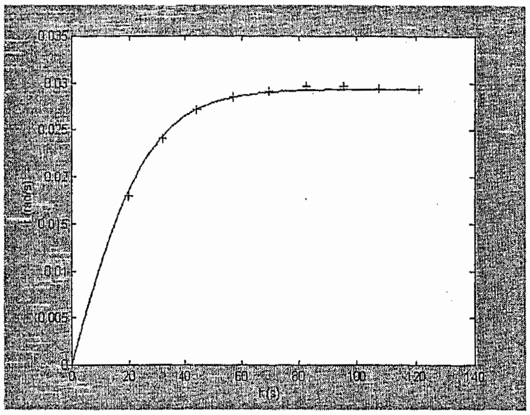

Fig. 4 Comparison between the experimental results (+) (Table 1) and the fit obtained (continuous line)

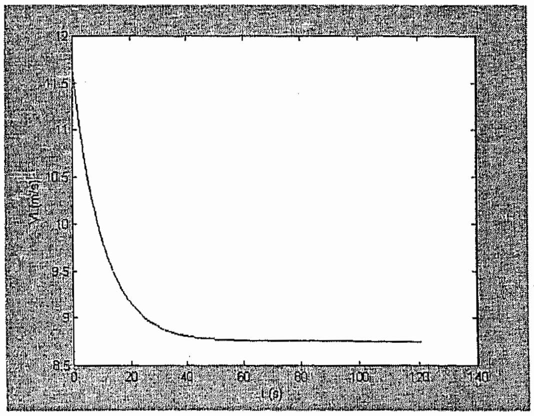

Fig. 5 Temporal variation on the surge speed during the turning circle test

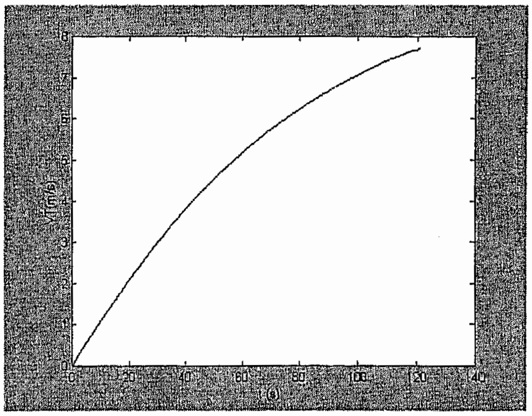

Fig. 6 Temporal variation on the sway speed during the turning circle test

4. Adaptive backstepping

The new recursive design known as adaptive backstepping (Krstić et al. 1992) is based on three techniques that differ in the construction of adaptation law:

1. Adaptive backstepping with overparameterization, when at each design step a new vector of adjustable parameters and the corresponding adaptation law are introduced (Kanellakopoulos et al. 1991). The controller is composed of a dynamic part, which is designed as a parameter update law where the static part of the controller is continuously adapted to new parameter estimates. This procedure has been chosen in the present paper.
2. Adaptive backstepping with modular identifiers, when a slight modification of the adaptive control allows the construction of estimation-based identifiers of unknown parameters (Kokotović 1992).
3. Adaptive backstepping with tuning functions, when at each design step a virtual adaptation law known as a tuning function is introduced, while the actual adaptation algorithm is defined at the final step in terms of all the previous tuning functions (Krstić et al. 1995).

Initially in the recursive procedure 1, the state x_{i+1} is treated as a virtual control for the subsystem consisting of the states $x_1, \ldots, x_i$. At each subsequent step, the designed subsystem will be increased by one equation. At the i- step, the ith-order subsystem is stabilized with respect to a Liapunov function V_i by the design of a stabilizing function α_1. The updating law of the adaptive control system that allows us to know the true values of the dynamic model and the control signal is designed at the final step. To implement the identification procedure based on backstepping, the following steps should be carried out:

4.1. Step 1

The following variables are introduced:

$$z_1 = x_1 \tag{8a}$$

$$z_2 = V \cdot \cos(x_2 - x_4) - \alpha_1 \tag{8b}$$

Where α_1 is used as a control to stabilize the z_1 subsystem, we choose the following Liapunov function candidate:

$$V_1 = \frac{1}{2} \cdot z_1^2 \tag{9}$$

In view of (9), (18a), and (7a), the derivative of V_1 satisfies the following:

$$\dot{V}_1 = z_1 \cdot \dot{z}_1 = z_1 \cdot \dot{x}_1 = z_1 \cdot [V \cdot \cos(x_2 - x_4)] \tag{10}$$

after considering (8b):

$$\dot{V}_1 = z_1 \cdot (z_2 + \alpha_1) \tag{11}$$

and choosing the stabilizing function α_1 as a simple static linear feedback law:

$$\alpha_1 = -K_1 \cdot z_1 = -K_1 \cdot x_1 = -K_1 \cdot (R - R_d) \tag{12}$$

which leads to

$$\dot{V}_1 = -K_1 \cdot z_1^2 + z_1 \cdot z_2 \tag{13}$$

Our purpose is to achieve a total Liapunov function for the entire system the temporal variation of which will be positive definite. The first term to verify this objective, the second term $z_1 \times z_2$ in (13), will be cancelled at the next step.

4.2. Step 2

It is necessary to consider that state x_3 is the control variable in the second equation (7b). The third backstepping variable z_3 is defined by the following equation:

$$z_3 = x_3 - \alpha_2 \tag{14}$$

where α_2 is a second stabilizing function used as a control to stabilize the (z_1, z_2) subsystem. In this step it is possible to meet the variation of the second error variable z_2, taking only its derivative in (8b), after considering (7b) and (7d), meeting

$$\dot{z}_2 = -V \cdot \left[x_3 - \frac{V}{x_1 + R_d} \cdot \sin(x_2 - x_4) \right] \cdot \quad \sin(x_2 - x_4) - \dot{\alpha}_1 \tag{15}$$

It is important to observe that the time derivative $\dot{\alpha}_1$ can be implemented analytically without a differentiator in the following manner:

$$\dot{\alpha}_1 = \frac{\partial \alpha_1}{\partial t} = \frac{\partial \alpha_1}{\partial R} \cdot \frac{\partial R}{\partial t} = -K_1 \cdot \dot{R} \tag{16}$$

In this step the initial Liapunov function is augmented with a quadratic term:

$$V_2 = V_1 + \frac{1}{2} \cdot z_2^2 \tag{17}$$

its temporal variation after considering equations (13) and (15):

$$\begin{aligned} \dot{V}_2 = &(-K_1 \cdot z_1^2 + z_1 \cdot z_2) - z_2 \cdot z_3 \cdot V \cdot \sin(x_2 - x_4) \\ &- z_2 \cdot \alpha_2 \cdot \sin(x_2 - x_4) + \frac{z_2 \cdot V^2}{x_1 + R_d} \cdot \sin^2(x_2 - x_4) \quad - z_2 \cdot \dot{\alpha}_1 \end{aligned} \tag{18}$$

If $\dot{V}_2$ must be negative definite in terms of z_1 and z_2, for this purpose the second stabilizing function α_2 is chosen as

$$\alpha_2 = \frac{1}{V \cdot \sin(x_2 - x_4)} \cdot \left[z_1 + K_2 \cdot z_2 + \frac{V^2 \cdot \sin^2(x_2 - x_4)}{x_1 + R_d} - \dot{\alpha}_1 \right] \tag{19}$$

In this way,

$$\dot{V}_2 = -K_1 \cdot z_1^2 - K_2 \cdot z_2^2 - z_2 \cdot z_3 \cdot V \cdot \sin(x_2 - x_4) \tag{20}$$

4.3. Step 3

In this last step we can meet the variation of the third error variable z_3 starting from equation (14).

$$\dot{z}_3 = \dot{x}_3 - \dot{\alpha}_2 = -\sum_{i=0}^{3} a_i \cdot x_3^i + c \cdot \delta - \dot{\alpha}_2 \tag{21}$$

The last Liapunov function utilized is

$$V_3 = V_2 + \frac{1}{2} \cdot z_3^2 + \frac{1}{2} \cdot \left[\sum_{i=0}^{3} \frac{1}{\gamma i} \cdot \tilde{a}_i^2 \right] \tag{22}$$

where $\tilde{a}_i$ ($i = 0 \cdots 3$) represents the estimation errors, differences between the true values a_i ($i = 0 \cdots 3$) and the estimation values $\hat{a}_i$ ($i = 0 \cdots 3$).

$$\dot{V}_3 = -K_1 \cdot z_1^2 - K_2 \cdot z_2^2 - z_2 \cdot z_3 \cdot V \cdot \sin(x_2 - x_4) + z_3 \cdot [-\sum_{i=0}^{3} \hat{a}_i \cdot x_3^i + c \cdot \delta - \dot{\alpha}_2] + \sum_{i=0}^{3} \frac{1}{\gamma i} \cdot \tilde{a}_i \cdot \dot{\tilde{a}}_i - \tilde{a}_i \cdot x_3^i \cdot z_3 \tag{23}$$

Using the control δ and an updating law to stabilize the entire system (z_1, z_2, z_3), we choose the control as

$$\delta = \frac{1}{c} \cdot [-K_3 \cdot z_3 + z_2 \cdot V \cdot \sin(x_2 - x_4) + \sum_{i=0}^{3} \hat{a}_i \cdot x_3^i + \dot{\alpha}_2] \tag{24}$$

With this choice the temporal variation of the control Liapunov function of the entire system is

$$\dot{V}_3 = -\sum_{i=1}^{3} K_i \cdot z_i^2 + \sum_{i=0}^{3} \tilde{a}_i \cdot \left[\frac{1}{\gamma i} \cdot \dot{\tilde{a}}_i - x_3^i \cdot z_3\right] \tag{25}$$

The terms that contain the parameter error are now eliminated with the update laws:

$$\frac{1}{\gamma i} \cdot \dot{\tilde{a}}_i = x_3^i \cdot z_3 \qquad (i = 0 \cdots 3) \tag{26}$$

This equation can be developed in the following terms:

$$i = 0 \qquad \dot{\tilde{a}}_0 = \gamma_0 \cdot z_3 \tag{27a}$$

$$i = 1 \qquad \dot{\tilde{a}}_2 = \gamma_1 \cdot x_3 \cdot z_3 = \gamma_1 \cdot r \cdot z_3 \tag{27b}$$

$$i = 2 \qquad \dot{\tilde{a}}_2 = \gamma_2 \cdot x_3^2 \cdot z_3 = \gamma_2 \cdot r^2 \cdot z_3 \tag{27c}$$

$$i = 3 \qquad \dot{\tilde{a}}_3 = \gamma_3 \cdot x_3^2 \cdot z_3 = \gamma_3 \cdot r^3 \cdot z_3 \tag{27d}$$

Previously described laws guarantee that

$$\dot{V}_3 = -\sum_{i=1}^{3} K_i \cdot z_i^2 \leq 0 \tag{28}$$

since V_3 is positive definite and radially unbounded and $\sum_{i=1}^{3} K_i \cdot z_i^2$ is positive definite, it follows from the LaSalle-Yoshizawa theorem (LaSalle 1968, Yoshizawa 1966) that in the (z_1, z_2, z_3) coordinates the equilibrium (0,0,0) is globally uniformly asymptotically stable (GUAS) and $\hat{a}_i(t)$, $i = 0 \cdots 3$ are globally uniformly bounded. In view of (8a), $z_1 = x_1$ go to zero asymptotically and $R \to R_d$ as $t \to \infty$. From (8b) for a V bounded and $\alpha_1 = -K_1 \cdot z_1$, this implies that x_2 also tends to zero in an asymptotic manner. From (14), x_3 tends toward α_2 (the second stabilizing function).

4.4. Error dynamics

For implementation purposes, it is convenient to obtain the error dynamics in the states previously introduced. The first dynamic is obtained to depart from (8a), (8b), and (12):

$$\dot{z}_1 = \dot{x}_1 = V \cdot \cos(x_2 - x_4) = z_2 + \alpha_1 = -K_1 \cdot z_1 + z_2 \tag{29}$$

The second equation can be obtained after considering equations (15) and (19):

$$\dot{z}_2 = -z_1 - K_2 \cdot z_2 - V \cdot z_3 \cdot \sin(x_2 - x_4) \tag{30}$$

The third equation is obtained from (21) and (24):

$$\dot{z}_3 = -\sum_{i=0}^{3} \tilde{a}_i \cdot x_3^i - K_3 \cdot z_3 + z_2 \cdot V \cdot \sin(x_2 - x_4) \tag{31}$$

Finally, the resulting error dynamics are obtained from equations (29), (30), and (31) and are written as:

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} -K_1 & 0 & 9 \\ 0 & -K_2 & 0 \\ 0 & 0 & -K_3 \end{bmatrix} \cdot \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & -V \cdot \sin(x_2 - x_4) \\ 0 & V \cdot \sin(x_2 - x_4) & 0 \end{bmatrix} \cdot \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -\sum_{i=0}^{3} \tilde{a}_i \cdot x_3^i \end{bmatrix} \tag{32}$$

4.5. Implementation of the identification procedure

The identification objective is based on the performance of the following tasks:

1. The performing of the turning circle test and the determination of the effective turning radius for a particular rudder angle and ship speed. In the ship that has been tested, the turning radius was R_d = 228 m, with a normal ballast condition and maximum speed of 22.68 knots. The initial value of the x_0 coordinate (see Fig. 3) was 188 m. It is worth pointing out that other important but nonfundamental features for this identification purpose were a transfer of 243 m, advance of 334 m, and tactical diameter of 416 m.
2. The application of the identification algorithm based on the implementation of equations (29), (30), and (31) and the updating laws (27a) to (27d) with a suitable simulation program, jointly with the stabilizing functions (12) and (19). The initial values of the estimated parameters $\hat{a}_i$ ($i = 0 \cdots 3$) are assumed to be 25% of the true values.
3. The application of some optimization criteria that let us reduce the z_i ($i = 0 \cdots 3$) variables to zero. The procedure is based on the Powell optimization algorithm (Darnell & Margolis 1990), a backward Euler integration method with a stepsize of 0.01 s. The gains that were obtained are shown in Table 4.

The numerical values determined by this identification procedure confirm the values previously determined by the alternative and heuristic one. The maximum relative error committed was less than 0.5%.

The identification algorithm can be extended to the determination of the coefficient c_0. There is an alternative form of computing this coefficient. The procedure starts from the steady value in the yaw rate temporal variation shown in Fig. 4. Under this condition, equation (3b) yields

Table 4 Gains obtained during the adaptation process

Gains	(p.u)
K_1	1.001
K_2	0.818
K_3	5.639
γ_0	−1
γ_1	−0.9986
γ_2	−10.1722
γ_3	−97,223.9
γ_4	-6.2783×10^6

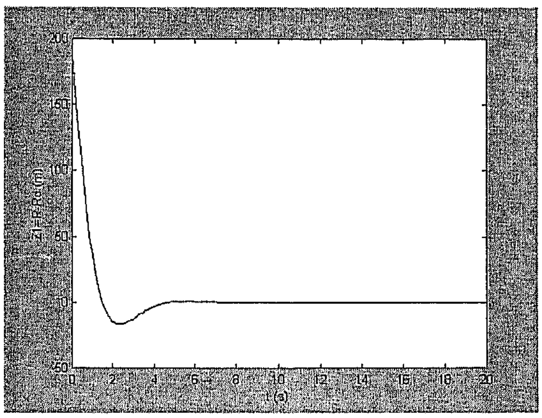

Fig. 7 Temporal variation of the Z_1 variable

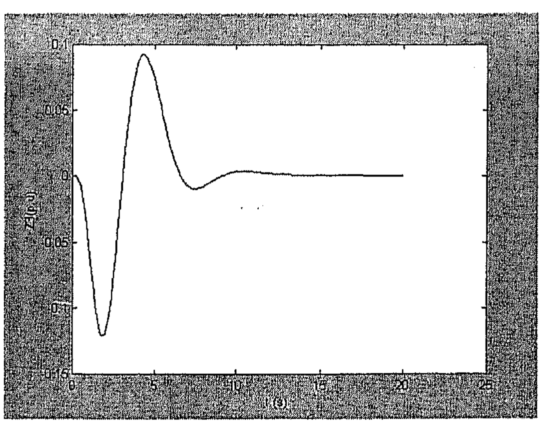

Fig. 8 Temporal variation of the Z_2 variable

$$c = -\frac{(\hat{a}_0 + \hat{a}_1 \cdot \bar{r} + \hat{a}_2 \cdot \bar{r}^2 + \hat{a}_3 \cdot \bar{r}^3)}{\delta} \tag{33}$$

where $\bar{r} = 2.93902 \cdot 10^{-2}$ rad/s, $\delta = 0.1745$ rad, and a_i ($i = 0 \cdots 3$), are obtained after the identification process. Its value coincides with the one indicated in Table 3.

Figures 7, 8, and 9 showed that state error variables z_1, z_2, and z_3 converge quickly toward the null values according to imposed requirements for the gain values K_1, K_2, K_3, K_3,γ_i ($i = 0 \cdots 3$) shown in Table 4.

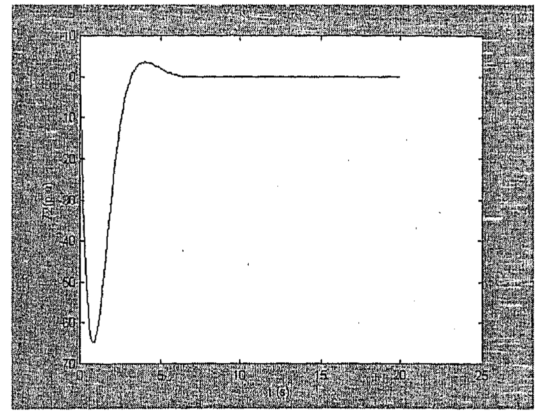

Fig. 9 Temporal variation of the Z_3 variable

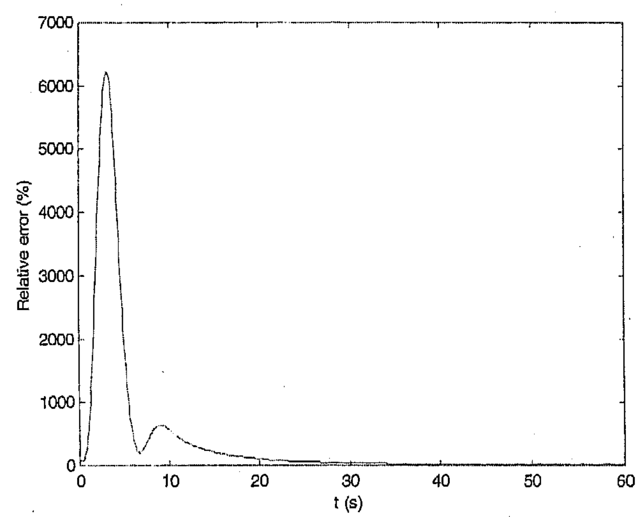

Fig. 10 Variation of estimation of the parameter a_0 (in percent of its true value) in the identification process

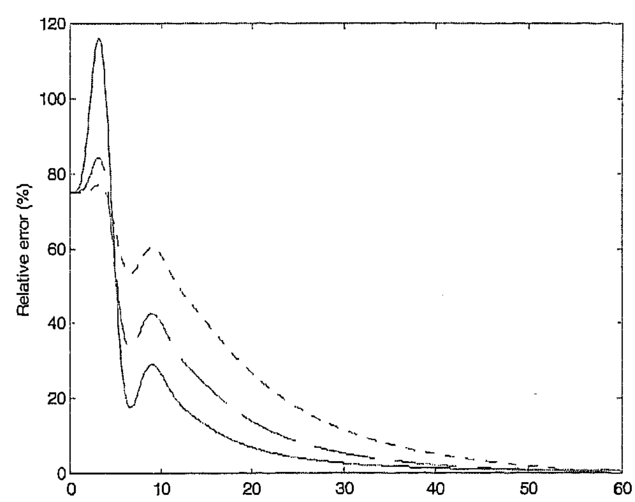

Fig. 11 Variation of the parameters (in percent of their true values) during the identification process (a_1, solid line; a_2, dashed line; a_3, dotted line)

5. Conclusions

The adaptive, identification, and tracking processes have been carried out. The procedure based on the backstepping design can perform this task starting from initial estimation values in a nonlinear ship characterized by a relatively complex nonlinear model capable of adjusting the ship's dynamics. The identification procedure needs only the value of the radius in the turning test trials. The identification procedure can be extended to the determination of the term c that multiplies to the rudder angle in the Norrbin equation that describes the ship motion.

Figures 10 and 11 show the temporal variations of the estimation of the parameters involved in equation (3b) produced by the estimation algorithms and the adaptation gains $\gamma_i (i = 0 \ldots 4)$ shown in Table 4. The initial estimations of the parameters in the dynamic equation (3b) account for 25% of the uncertainties in these parameters.

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